2-1 Power and Radical Functions

Graph and analyze each function. Describe the domain, range, intercepts, end behavior, continuity, and where the function is increasing or decreasing.

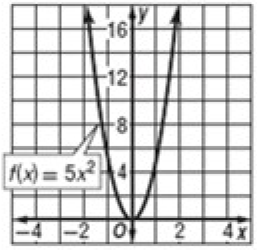
1. $f(x) = 5x^2$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−3	−2	−1	0	1	2	3
$f(x)$	45	20	5	0	5	20	45

Use these points to construct a graph.



The function is a monomial with an even degree and a positive value for a.

D = $(-\infty, \infty)$, R = $[0, \infty)$; intercept: 0; $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = \infty$; continuous for all real numbers; decreasing: $(-\infty, 0)$; increasing: $(0, \infty)$

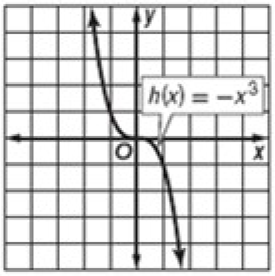
3. $h(x) = -x^3$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−3	−2	−1	0	1	2	3
$f(x)$	27	8	1	0	−1	−8	−27

Use these points to construct a graph.



The function is a monomial with an odd degree and a negative value for a.

D = $(-\infty, \infty)$, R = $(-\infty, \infty)$; intercept: 0; $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = -\infty$; continuous for all real numbers; decreasing: $(-\infty, \infty)$

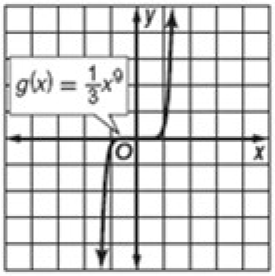
5. $g(x) = \frac{1}{3}x^9$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−3	−2	−1	0	1	2	3
$f(x)$	−6561	−170.7	−0.3	0	0.3	170.7	6561

Use these points to construct a graph.



The function is a monomial with an odd degree and a positive value for a.

D = $(-\infty, \infty)$, R = $(-\infty, \infty)$; intercept: 0; $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$; continuous for all real numbers; increasing: $(-\infty, \infty)$

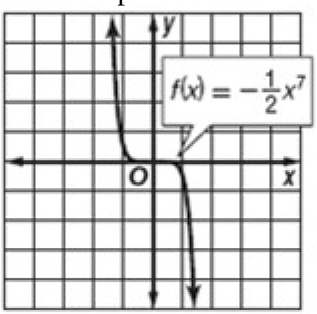
7. $f(x) = -\frac{1}{2}x^7$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−3	−2	−1	0	1	2	3
$f(x)$	1093.5	64	0.5	0	−0.5	−64	−1093.5

Use these points to construct a graph.



The function is a monomial with an odd degree and a negative value for a.

D = $(-\infty, \infty)$, R = $(-\infty, \infty)$; intercept: 0; $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = -\infty$; continuous for all real numbers; decreasing: $(-\infty, \infty)$

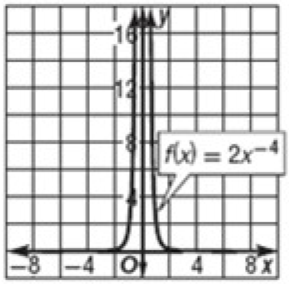
9. $f(x) = 2x^{-4}$

SOLUTION:

Evaluate the function for several *x*-values in its domain.

x	−3	−2	−1	0	1	2	3
f(x)	0.025	0.125	2		2	0.125	0.025

Use these points to construct a graph.



Since the power is negative, the function will be undefined at $x = 0$.

$D = (-\infty, 0) \cup (0, \infty)$, $R = (0, \infty)$; no intercepts; $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = 0$; infinite discontinuity at $x = 0$; increasing: $(-\infty, 0)$; decreasing: $(0, \infty)$

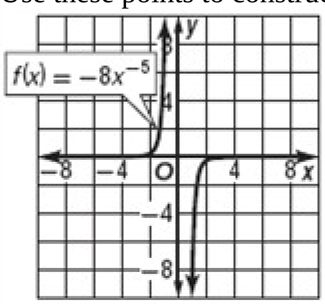
11. $f(x) = -8x^{-5}$

SOLUTION:

Evaluate the function for several *x*-values in its domain.

x	−3	−2	−1	0	1	2	3
f(x)	0.03	0.25	8		−8	−0.25	−0.03

Use these points to construct a graph.



Since the power is negative, the function will be undefined at $x = 0$.

$D = (-\infty, 0) \cup (0, \infty)$, $R = (-\infty, 0) \cup (0, \infty)$; no intercepts; $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = 0$; infinite discontinuity at $x = 0$; increasing: $(-\infty, 0)$ and $(0, \infty)$

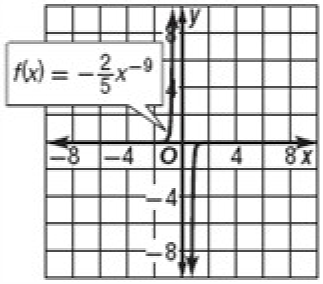
13. $f(x) = -\frac{2}{5}x^{-9}$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−1.5	−1	−0.5	0	0.5	1	1.5
$f(x)$	0.01	0.4	204.8		−204.8	−0.4	−0.01

Use these points to construct a graph.



Since the power is negative, the function will be undefined at $x = 0$.

D = $(-\infty, 0) \cup (0, \infty)$, R = $(-\infty, 0) \cup (0, \infty)$; no intercepts; $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = 0$; infinite discontinuity at $x = 0$; increasing: $(-\infty, 0)$ and $(0, \infty)$

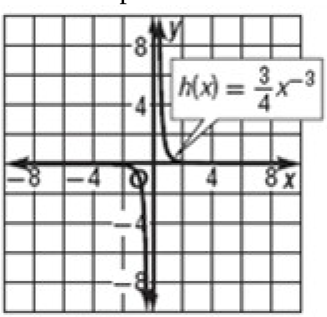
15. $h(x) = \frac{3}{4}x^{-3}$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−1.5	−1	−0.5	0	0.5	1	1.5
$f(x)$	−0.22	−0.75	−6		6	0.75	0.22

Use these points to construct a graph.



Since the power is negative, the function will be undefined at $x = 0$.

D = $(-\infty, 0) \cup (0, \infty)$, R = $(-\infty, 0) \cup (0, \infty)$; no intercepts; $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = 0$; infinite discontinuity at $x = 0$; decreasing: $(-\infty, 0)$ and $(0, \infty)$

17. **GEOMETRY** The volume of a sphere is given by $V(r) = \frac{4}{3}\pi r^3$, where r is the radius.

a. State the domain and range of the function.
b. Graph the function.

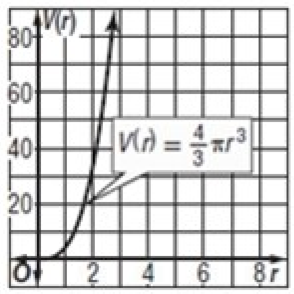
SOLUTION:

a. The radius of a sphere cannot have a negative length. The radius also cannot be 0 because then the object would fail to be a sphere. Thus, D = $(0, \infty)$, R = $(0, \infty)$

b. Evaluate the function for several x-values in its domain.

x	0.5	1	1.5	2	2.5	3	3.5
$f(x)$	0.5	4.2	14.1	33.5	65.5	113.1	179.6

Use these points to construct a graph.



Graph and analyze each function. Describe the domain, range, intercepts, end behavior, continuity, and where the function is increasing or decreasing.

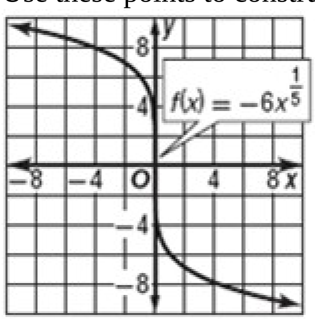
19. $f(x) = -6x^{\frac{1}{5}}$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−6	−4	−2	0	2	4	6
$f(x)$	8.6	7.9	6.9	0	−6.9	−7.9	−8.6

Use these points to construct a graph.



D = $(-\infty, \infty)$, R = $(-\infty, \infty)$; intercept: 0; $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = -\infty$; continuous for all real numbers; decreasing: $(-\infty, \infty)$

2-1 Power and Radical Functions

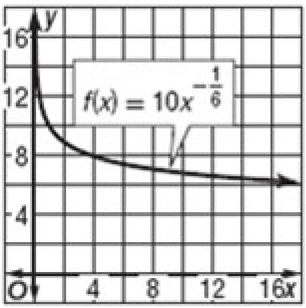
21. $f(x) = 10x^{-\frac{1}{6}}$

SOLUTION:

Evaluate the function for several *x*-values in its domain.

x	1	2	3	4	5	6	7
f **(*x*)**	10.0	8.9	8.3	7.9	7.6	7.4	7.2

Use these points to construct a graph.



Since the denominator of the power is even and the power is negative, the domain must be restricted to positive values.

D = (0, ∞), R = (0, ∞); no intercepts; $\lim_{x \to \infty} f(x) = 0$; continuous on (0, ∞); decreasing: (0, ∞)

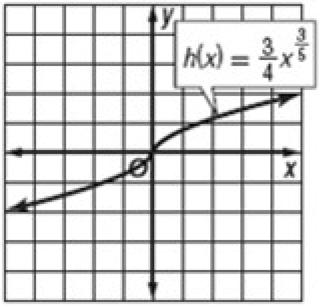
23. $h(x) = \frac{3}{4}x^{\frac{3}{5}}$

SOLUTION:

Evaluate the function for several *x*-values in its domain.

x	−3	−2	−1	0	1	2	3
f **(*x*)**	−1.45	−1.13	−0.75	0	0.75	1.13	1.45

Use these points to construct a graph.



D = (−∞, ∞), R = (−∞, ∞); intercept: 0; $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$; continuous for all real numbers; increasing: (−∞, ∞)

2-1 Power and Radical Functions

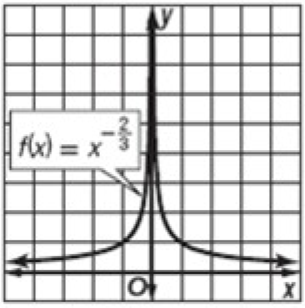
25. $f(x) = x^{-\frac{2}{3}}$

SOLUTION:

Evaluate the function for several *x*-values in its domain.

x	−3	−2	−1	0	1	2	3
***f* (*x*)**	0.48	0.63	1		1	0.63	0.48

Use these points to construct a graph.



Since the power is negative, the function will be undefined at $x = 0$.

D = $(-\infty, 0) \cup (0, \infty)$, R = $(0, \infty)$; no intercepts; $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = 0$; infinite discontinuity at $x = 0$; increasing: $(-\infty, 0)$; decreasing: $(0, \infty)$

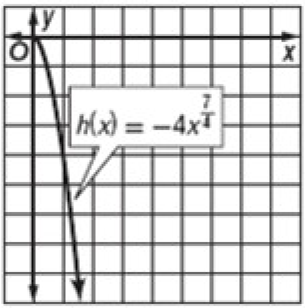
27. $h(x) = -4x^{\frac{7}{4}}$

SOLUTION:

Evaluate the function for several *x*-values in its domain.

x	0	0.5	1	1.5	2	2.5	3
***f* (*x*)**	0	−1.2	−4	−8.1	−13.5	−19.9	−27.4

Use these points to construct a graph.



Since the denominator of the power is even, the domain must be restricted to nonnegative values.

D = $[0, \infty)$, R = $(-\infty, 0]$; intercept: 0; $\lim_{x \to \infty} f(x) = -\infty$; continuous on $[0, \infty)$; decreasing: $(0, \infty)$

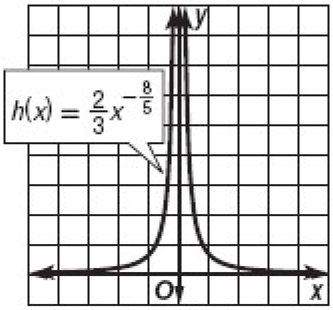
29. $h(x) = \frac{2}{3}x^{-\frac{8}{5}}$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−3	−2	−1	0	1	2	3
f **(*x*)**	0.11	0.22	0.67		0.67	0.22	0.11

Use these points to construct a graph.



Since the power is negative, the function will be undefined at $x = 0$.

D = $(-\infty, 0) \cup (0, \infty)$, R = $(0, \infty)$; no intercepts; $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = 0$; infinite discontinuity at $x = 0$;

increasing: $(-\infty, 0)$; decreasing: $(0, \infty)$

2-1 Power and Radical Functions

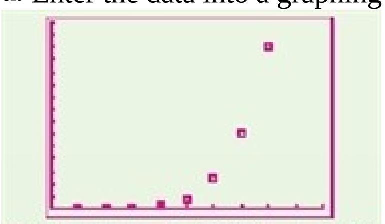
Complete each step.
a. Create a scatter plot of the data.
b. Determine a power function to model the data.
c. Calculate the value of each model at x = 30.

31.

x	y
1	1
2	32
3	360
4	2000
5	7800
6	25,000
7	60,000
8	130,000

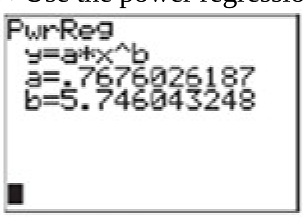
SOLUTION:

a. Enter the data into a graphing calculator and create a scatter plot.



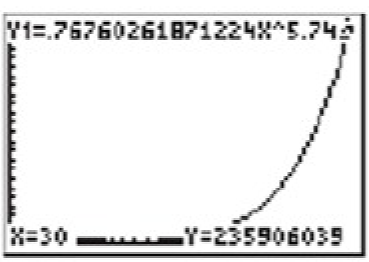
[0, 10] scl: 1 by [0, 150000] scl: 10000

b. Use the power regression function on the graphing calculator to find values for a and b.



$y = 0.77x^{5.75}$.

c. Graph the regression equation using a graphing calculator. To calculate x = 30, use the **CALC** function on the graphing calculator.



[0, 30] scl: 1 by [0, 240, 000, 000] scl: 10, 000, 000

The value of the model at x = 30 is about 235,906,039.

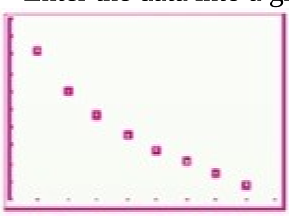
33. **WEATHER** The wind chill temperature is the apparent temperature felt on exposed skin, taking into account the effect of the wind. The table shows the wind chill temperature produced at winds of various speeds when the actual temperature is 50ºF.

Wind Speed (mph)	Wind Chill (°F)
5	48.22
10	46.04
15	44.64
20	43.60
25	42.76
30	42.04
35	41.43
40	40.88

a. Create a scatter plot of the data.
b. Determine a power function to model the data.
c. Use the function to predict the wind chill temperature when the wind speed is 65 miles per hour.

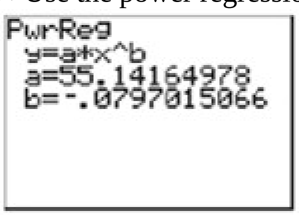
SOLUTION:

a. Enter the data into a graphing calculator and create a scatter plot.



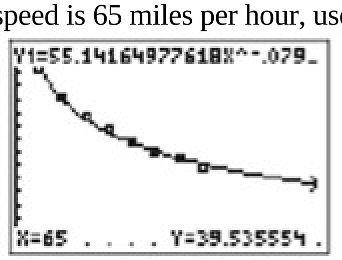
[0, 45] scl: 5 by [40, 50] scl: 1

b. Use the power regression function on the graphing calculator to find values for *a* and *b*.



$f(x) = 55.14x^{-0.0797}$.

c. Graph the regression equation using a graphing calculator. To predict the wind chill temperature when the wind speed is 65 miles per hour, use the **CALC** function on the graphing calculator. Let $x = 65$.



[0, 65] scl: 5 by [35, 50] scl: 1

The wind chill temperature when the wind speed is 65 miles per hour is about 39.54°F

2-1 Power and Radical Functions

Graph and analyze each function. Describe the domain, range, intercepts, end behavior, continuity, and where the function is increasing or decreasing.

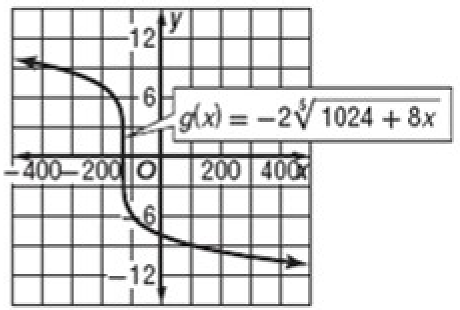
35. $g(x) = -2\sqrt[5]{1024 + 8x}$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−300	−200	−100	0	100	200	300
$f(x)$	8.5	7.1	−5.9	−8	−9	−9.7	−10.2

Use these points to construct a graph.



Solve for x when $g(x)$ is 0 to find the x-intercept.

$$0 = -2\sqrt[5]{51024 + 8x}$$
$$0 = \sqrt[5]{51024 + 8x}$$
$$0 = 1024 + 8x$$
$$-1024 = 8x$$
$$-128 = x$$

D = $(-\infty, \infty)$, R = $(-\infty, \infty)$; x-intercept: −128, y-intercept: −8; $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = -\infty$; continuous for all real numbers; decreasing: $(-\infty, \infty)$

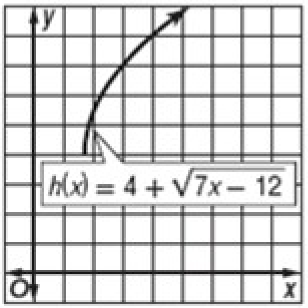
37. $h(x) = 4 + \sqrt{7x - 12}$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	$\frac{12}{7}$	2	3	4	5	6	7
$f(x)$	4	5.4	7	8	8.8	9.5	10.1

Use these points to construct a graph.



Since it is an even-degree radical function, the domain is restricted to nonnegative values for the radicand, $7x - 12$. Solve for x when the radicand is 0 to find the restriction on the domain.

$$0 = 7x - 12$$
$$12 = 7x$$
$$\frac{12}{7} = x$$

$D = \left[\frac{12}{7}, \infty\right)$, R = [4, ∞); no intercepts; $\lim_{x \to \infty} f(x) = \infty$; continuous on $\left[\frac{12}{7}, \infty\right)$; increasing: $\left(\frac{12}{7}, \infty\right)$

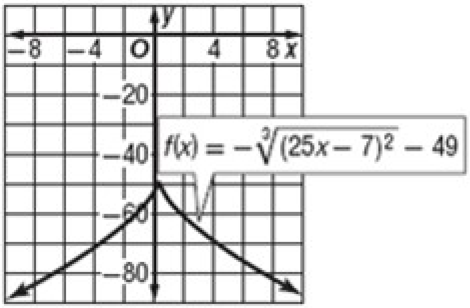
39. $f(x) = -\sqrt[3]{(25x-7)^2} - 49$

SOLUTION:

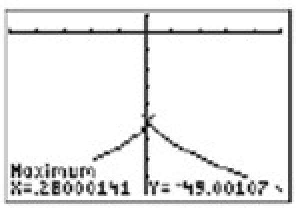
Evaluate the function for several *x*-values in its domain.

x	−6	−4	−2	0	2	4	6
***f*(*x*)**	−78.1	−71.54	−63.81	−52.66	−61.27	−69.53	−76.35

Use these points to construct a graph.



Use the maximum function or the trace function on a graphing calculator to approximate the maximum value of *f* (*x*) at (0.28, −49).



[−10, 10] scl: 2 by [−90, 10] scl: 10

The range is restricted to values less than or equal to −49. Also, there is a turning point at *x* = 0.28.

D = (−∞ , ∞), R = (−∞ , −49.00]; *x*-intercept: none, *y*-intercept: −52.66;

$\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = -\infty$; continuous for all real numbers; increasing: (−∞ , 0.28); decreasing: (0.28, ∞)

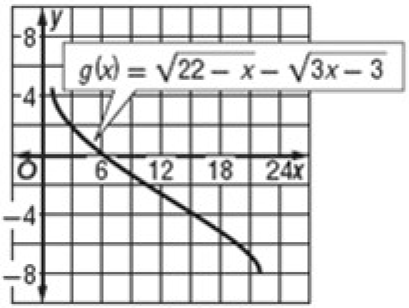
41. $g(x) = \sqrt{22x} - \sqrt{3x3}$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	1	3	6	12	15	18	22
$f(x)$	4.58	1.91	0.13	−2.58	−3.84	−5.14	−7.94

Use these points to construct a graph.



Since $g(x)$ includes two even-degree radicals, the domain is restricted to nonnegative values for each radicand, $22 - x$ and $3x - 3$. Solve for x when each radicand is greater than or equal to 0 to find the restrictions on the domain.

$$\begin{aligned} 0 &\le 22 - x & 0 &\le 3x - 3 \\ -22 &\le -x & 3 &\le 3x \\ 22 &\ge x & 1 &\le x \end{aligned}$$

Thus, x must be $1 \le x \le 22$. Substitute these values for x to find the restrictions on the range.

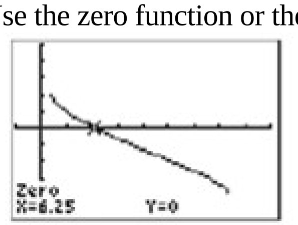
$x = 1$

$$\begin{aligned} g(x) &= \sqrt{22 - x} - \sqrt{3x - 3} \\ &= \sqrt{22 - 1} - \sqrt{3(1) - 3} \\ &= \sqrt{21} \end{aligned}$$

$x = 22$

$$\begin{aligned} g(x) &= \sqrt{22 - x} - \sqrt{3x - 3} \\ &= \sqrt{22 - 22} - \sqrt{3(22) - 3} \\ &= -\sqrt{63} \end{aligned}$$

Use the zero function or the trace function on a graphing calculator to approximate the x-intercept at (6.25, 0).



[−3, 27] scl: 3 by [−10, 10] scl: 2

D = [1, 22], R = $[-\sqrt{63}, \sqrt{21}]$; x-intercept: 6.25; continuous on [1, 22]; decreasing: (1, 22)

43\. **AGRICULTURAL SCIENCE** The net energy NE_m required to maintain the body weight of beef cattle, in megacalories (Mcal) per day, is estimated by the formula $NE_m = 0.077\sqrt[4]{m^3}$, where *m* is the animal's mass in kilograms. One megacalorie is equal to one million calories.

a. Find the net energy per day required to maintain a 400-kilogram steer.

b. If 0.96 megacalorie of energy is provided per pound of whole grain corn, how much corn does a 400-kilogram steer need to consume daily to maintain its body weight?

SOLUTION:

a. Substitute *m* = 400.

$$\begin{aligned} NE_m &= 0.077\sqrt[4]{m^3} \\ &= 0.077\sqrt[4]{400^3} \\ &= 6.89 \end{aligned}$$

The net energy per day required to maintain a 400-kilogram steer is approximately 6.89 Mcal.

b. Divide 6.89 Mcal by the 0.96 Mcal found in a pound of whole grain corn to find the total amount of corn necessary to maintain a 400-kilogram steer.

$$= \frac{6.89}{0.98}$$
$$= 7.18$$

It will take about 7.18 pounds of corn to maintain a 400-kilogram steer.

Solve each equation.

45\. $0.5x = \sqrt{4-3x} + 2$

SOLUTION:

$$\begin{aligned} 0.5x &= \sqrt{4-3x} + 2 \\ 0.5x - 2 &= \sqrt{4-3x} \\ 0.25x^2 - 2x + 4 &= 4 - 3x \\ 0.25x^2 + x &= 0 \\ x(0.25x + 1) &= 0 \\ 0.25x + 1 = 0 \quad &\text{or} \quad x = 0 \\ 0.25x &= -1 \\ x &= -4 \end{aligned}$$

Since the each side of the equation was raised to a power, check the solutions in the original equation.

$x = 0$

$$\begin{aligned} 0.5x &= \sqrt{4-3x} + 2 \\ 0.5(0) &= \sqrt{4-3(0)} + 2 \\ 0 &\neq 4 \end{aligned}$$

$x = -4$

$$\begin{aligned} 0.5x &= \sqrt{4-3x} + 2 \\ 0.5(-4) &= \sqrt{4-3(-4)} + 2 \\ -2 &\neq 6 \end{aligned}$$

Neither value for *x* is a solution for the original equation. Thus, there is no solution.

47. $\sqrt{(2x-5)3} - 10 = 17$

SOLUTION:

$$\begin{aligned}\sqrt{(2x-5)} - 10 &= 17\\ \sqrt{(2x-5)^3} &= 27\\ (2x-5)^3 &= 729\\ 2x-5 &= 9\\ 2x &= 14\\ x &= 7\end{aligned}$$

Since the each side of the equation was raised to a power, check the solution in the original equation.

$$\begin{aligned}\sqrt{(2x-5)^3} - 10 &= 17\\ \sqrt{(2(7)-5)^3} - 10 &= 17\\ 17 &= 17\end{aligned}$$

The solution is $x = 7$.

49. $x = \sqrt{2x-4} + 2$

SOLUTION:

$$\begin{aligned}x &= \sqrt{2x-4} + 2\\ x-2 &= \sqrt{2x-4}\\ x^2 - 4x + 4 &= 2x - 4\\ x^2 - 6x + 8 &= 0\\ (x-2)(x-4) &= 0\\ x-2 = 0 \quad &or \quad x-4 = 0\\ x = 2 \quad & \quad\quad x = 4\end{aligned}$$

Since the each side of the equation was raised to a power, check the solutions in the original equation.

$x = 2$

$$\begin{aligned}x &= \sqrt{2x-4} + 2\\ 2 &= \sqrt{2(2)-4} + 2\\ 2 &= 2\end{aligned}$$

$x = 4$

$$\begin{aligned}x &= \sqrt{2x-4} + 2\\ 4 &= \sqrt{2(4)-4} + 2\\ 4 &= 4\end{aligned}$$

The solutions are $x = 2$ and $x = 4$.

51. $x = 5 + \sqrt{x+1}$

SOLUTION:

$$\begin{aligned} x &= 5+\sqrt{x+1} \\ x-5 &= \sqrt{x+1} \\ x^2-10x+25 &= x+1 \\ x^2-11x+24 &= 0 \\ (x-8)(x-3) &= 0 \\ x-8 &= 0 \quad \text{or} \quad x-3=0 \\ x &= 8 \qquad\qquad x = 3 \end{aligned}$$

Since the each side of the equation was raised to a power, check the solutions in the original equation.

$x = 8$

$$\begin{aligned} x &= 5+\sqrt{x+1} \\ 8 &= 5+\sqrt{8+1} \\ 8 &= 8 \end{aligned}$$

$x = 3$

$$\begin{aligned} x &= 5+\sqrt{x+1} \\ 3 &= 5+\sqrt{3+1} \\ 3 &\neq 7 \end{aligned}$$

One solution checks and the other solution does not. Therefore, the solution is $x = 8$.

53. $\sqrt{4x-40} = -20$

SOLUTION:

$$\begin{aligned} \sqrt{4x-40} &= 20 \\ 4x-40 &= 400 \\ 4x &= 440 \\ x &= 110 \end{aligned}$$

Since the each side of the equation was raised to a power, check the solution in the original equation.

$$\begin{aligned} \sqrt{4x-40} &= -20 \\ \sqrt{4(110)-40} &= -20 \\ 20 &\neq -20 \end{aligned}$$

There is no solution.

55. $7+\sqrt[5]{1054-3x}=11$

SOLUTION:

$$\begin{aligned}7+\sqrt[5]{1054-3x}&=11\\ \sqrt[5]{1054-3x}&=4\\ 1054-3x&=1024\\ -3x&=-30\\ x&=10\end{aligned}$$

Since the each side of the equation was raised to a power, check the solution in the original equation.

$$\begin{aligned}7+\sqrt[5]{1054-3x}&=11\\ 7+\sqrt[5]{1054-3(10)}&=11\\ 11&=11\end{aligned}$$

The solution is $x = 10$.

Determine whether each function is a monomial function given that *a* and *b* are positive integers. Explain your reasoning.

57. $G(x) = -2ax^4$

SOLUTION:

Yes; sample answer: The function follows the form $f(x) = ax^n$, where n is a positive integer. In this case, $a = -2a$ and $n = 4$.

59. $y = \frac{7}{3}t^{ab}$

SOLUTION:

Yes; sample answer: The function follows the form $f(x) = ax^n$, where n is a positive integer. In this case, $a = \frac{7}{3}$ and $n = ab$.

61. $y = 4abx^{-2}$

SOLUTION:

No; sample answer: The function is not a monomial function because the exponent for x is negative.

Solve each inequality.

63. $\sqrt[5]{1040+8x} \geq 4$

SOLUTION:

$$\begin{aligned} \sqrt[5]{1040+8x} &\geq 4 \\ 1040+8x &\geq 1024 \\ 8x &\geq -16 \\ x &\geq -2 \end{aligned}$$

Since the each side of the equation was raised to a power, check a solution in the original equation.

$x = -1$

$$\begin{aligned} \sqrt[5]{1040+8x} &\geq 4 \\ \sqrt[5]{1040+8(-1)} &\geq 4 \\ 4.006 &\geq 4 \end{aligned}$$

The solution is $x \geq -2$.

65. $(1-4x)^{\frac{3}{2}} \geq 125$

SOLUTION:

$$\begin{aligned} (1-4x)^{\frac{3}{2}} &\geq 125 \\ 1-4x &\geq 25 \\ -4x &\geq 24 \\ x &\leq -6 \end{aligned}$$

Since the each side of the equation was raised to a power, check a solution in the original equation.

$x = -7$

$$\begin{aligned} (1-4x)^{\frac{3}{2}} &\geq 125 \\ [1-4(-7)]^{\frac{3}{2}} &\geq 125 \\ 156.17 &\geq 125 \end{aligned}$$

Since the denominator of the exponent is even, the function must be checked for restrictions on the domain. The radicand, $1 - 4x$, must be greater than or equal to 0. Solve $1 - 4x \geq 0$ for x.

$$\begin{aligned} 1-4x &\geq 0 \\ -4x &\geq -1 \\ x &\leq \frac{1}{4} \end{aligned}$$

The solution accounts for this restriction. So, the solution is $x \leq -6$.

67. $(19-4x)^{\frac{5}{3}}-12\leq-13$

SOLUTION:

$$\begin{aligned}(19-4x)^{\frac{5}{3}}-12&\leq-13\\(19-4x)^{\frac{5}{3}}&\leq-1\\19-4x&\leq-1\\-4x&\leq-20\\x&\geq5\end{aligned}$$

Since the each side of the equation was raised to a power, check a solution in the original equation.
$x = 6$

$$\begin{aligned}(19-4x)^{\frac{5}{3}}-12&\leq-13\\[19-4(6)]^{\frac{5}{3}}-12&\leq-13\\-26.62&\leq-13\end{aligned}$$

The solution is $x \geq 5$.

69. **CHEMISTRY** Boyle's Law states that, at constant temperature, the pressure of a gas is inversely proportional to its volume. The results of an experiment to explore Boyle's Law are shown.

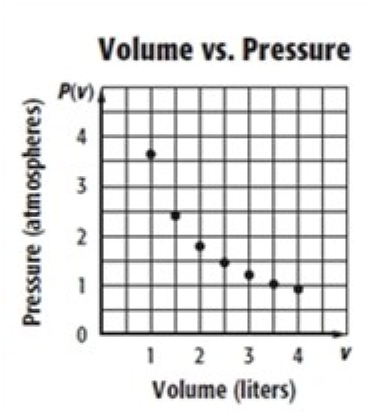
Volume (liters)	Pressure (atmospheres)
1.0	3.65
1.5	2.41
2.0	1.79
2.5	1.46
3.0	1.21
3.5	1.02
4.0	0.92

a. Create a scatter plot of the data.
b. Determine a power function to model the pressure P as a function of volume v.
c. Based on the information provided in the problem statement, does the function you determined in part **b** make sense? Explain.
d. Use the model to predict the pressure of the gas if the volume is 3.25 liters.
e. Use the model to predict the pressure of the gas if the volume is 6 liters.

SOLUTION:

a.

2-1 Power and Radical Functions

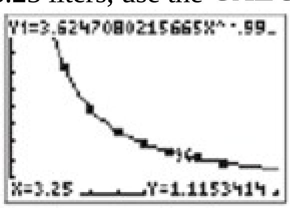


b. Use the power regression function on the graphing calculator to find values for *a* and *n*.

$P(v) = 3.62v^{-1}$

c. Sample answer: Yes; the problem states that the volume and pressure are inversely proportional, and in the power function, the exponent of the volume variable is −1.

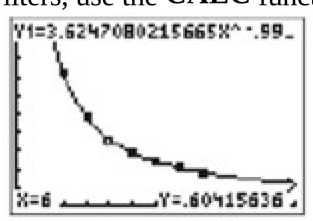
d. Graph the regression equation using a graphing calculator. To predict the pressure of the gas if the volume is 3.25 liters, use the **CALC** function on the graphing calculator. Let $v = 3.25$.



[0, 5] scl: 0.5 by [0, 5] scl: 0.5

The pressure of 3.25 liters of the gas is about 1.12 atmospheres.

e. Graph the regression equation using a graphing calculator. To predict the pressure of the gas if the volume is 6 liters, use the **CALC** function on the graphing calculator. Let $v = 6$.



[0, 6] scl: 0.5 by [0, 5] scl: 0.5

The pressure of 6 liters of the gas is about 0.60 atmospheres.

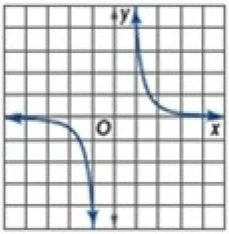
Without using a calculator, match each graph with the appropriate function.

a. $f(x) = \frac{1}{2}\sqrt[4]{3x^5}$

b. $g(x) = \frac{2}{3}x^6$

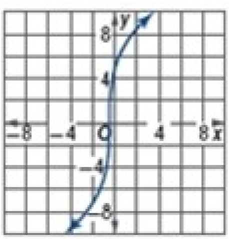
c. $h(x) = 4x^{-3}$

d. $p(x) = 5\sqrt[3]{2x+1}$

71. 

SOLUTION:

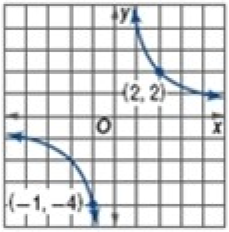
There is an infinite discontinuity at $x = 0$. This indicates that the power is negative. The equation that matches this description is $h(x) = 4x^{-3}$.
The answer is c.

73. 

SOLUTION:

The end behavior and the continuity of the graph indicate that it is a radical function with an odd value for n. The equation that matches this description is $p(x) = 5\sqrt[3]{2x+1}$.
The answer is d.

Use the points provided to determine the power function represented by the graph.

75. 

SOLUTION:

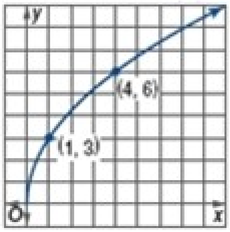
The function is undefined at $x = 0$, therefore; it is a power function of the form $f(x) = ax^{-n}$ or $f(x) = a\frac{1}{x^n}$, where a is a real constant and n is a natural number. Start by assuming $n = 1$ and solve for a by substituting a set of points (2, 2) for x and y.

$$f(x) = a\frac{1}{x^n}$$
$$2 = a\frac{1}{2^1}$$
$$2 = a\frac{1}{2}$$
$$4 = a$$

When $n = 1$, $f(x) = 4 \cdot \frac{1}{x}$ or $\frac{4}{x}$ for the point (2, 2). If this power function is true for the second point (−1, −4), then $f(x)$ can represent the graph.

$$f(x) = \frac{4}{x}$$
$$-4 = \frac{4}{-1}$$
$$-4 = -4$$

Since $f(x)$ is true for (−1, −4), $f(x) = \frac{4}{x}$ or $4x^{-1}$ is a power function for the graph.

77. 

SOLUTION:

The restricted domain of the graph indicates that it is a power function of the form $f(x) = ax^{\frac{1}{n}}$, where *a* is a real number and *n* is an even integer. Start by assuming *n* = 2 and solve for *a* by substituting a set of points (1, 3) for *x* and *y*.

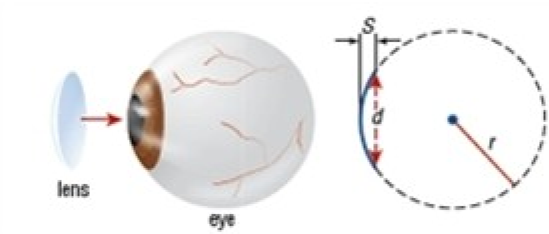
$$f(x) = ax^{\frac{1}{n}}$$
$$3 = a(1)^{\frac{1}{2}}$$
$$3 = a$$

When *n* =2, $f(x) = 3x^{\frac{1}{2}}$ for the point (1, 3). If this power function is true for the second point (4, 6), then *f* (*x*) can represent the graph.

$$f(x) = 3x^{\frac{1}{2}}$$
$$6 = 3(4)^{\frac{1}{2}}$$
$$6 = 6$$

Since *f* (*x*) is true for (4, 6), $f(x) = 3x^{\frac{1}{2}}$ is a power function for the graph.

79. **OPTICS** A contact lens with the appropriate depth ensures proper fit and oxygen permeation. The depth of a lens can be calculated using the formula $S = r - \sqrt{r^2 - \left(\frac{d}{2}\right)^2}$, where *S* is the depth, *r* is the radius of curvature, and *d* is the diameter, with all units in millimeters.



a. If the depth of the contact lens is 1.15 millimeters and the radius of curvature is 7.50 millimeters, what is the diameter of the contact lens?
b. If the depth of the contact lens is increased by 0.1 millimeter and the diameter of the lens is 8.2 millimeters, what radius of curvature would be required?
c. If the radius of curvature remains constant, does the depth of the contact lens increase or decrease as the diameter increases?

SOLUTION:

a. Substitute S = 1.15 and r = 7.50 into $S = r - \sqrt{r^2 - \left(\frac{d}{2}\right)^2}$ and solve for d.

$$S = r - \sqrt{r^2 - \left(\frac{d}{2}\right)^2}$$

$$1.15 = 7.50 - \sqrt{7.50^2 - \left(\frac{d}{2}\right)^2}$$

$$-6.35 = -\sqrt{7.50^2 - \left(\frac{d}{2}\right)^2}$$

$$6.35 = \sqrt{7.50^2 - \left(\frac{d}{2}\right)^2}$$

$$40.3225 = 7.50^2 - \left(\frac{d}{2}\right)^2$$

$$-15.9275 = -\left(\frac{d}{2}\right)^2$$

$$15.9275 = \left(\frac{d}{2}\right)^2$$

$$3.99 = \frac{d}{2}$$

$$7.98 = d$$

Since the each side of the equation was raised to a power, check the solution in the original equation.

$$S = r - \sqrt{r^2 - \left(\frac{d}{2}\right)^2}$$

$$1.15 = 7.50 - \sqrt{7.50^2 - \left(\frac{7.98}{2}\right)^2}$$

$$1.15 = 1.15$$

The diameter of the contact is 7.98 mm.

b. Substitute S = 1.15 + 0.1 or 1.25 and d = 8.2 into $S = r - \sqrt{r^2 - \left(\frac{d}{2}\right)^2}$ and solve for r.

$$S = r - \sqrt{r^2 - \left(\frac{d}{2}\right)^2}$$

$$1.25 = r - \sqrt{r^2 - \left(\frac{8.2}{2}\right)^2}$$

$$1.25 = r - \sqrt{r^2 - 16.81}$$

$$r - 1.25 = \sqrt{r^2 - 16.81}$$

$$r^2 - 2.50r + 1.5625 = r^2 - 16.81$$

$$-2.50r = -18.3725$$

$$r = 7.35$$

Since the each side of the equation was raised to a power, check the solution in the original equation.

$$S = r - \sqrt{r^2 - \left(\frac{d}{2}\right)^2}$$

$$1.25 = 7.35 - \sqrt{7.35^2 - \left(\frac{8.2}{2}\right)^2}$$

$$1.25 = 1.25$$

The radius of curvature required is 7.35 mm.

c. As d increases, the value of $\left(\frac{d}{2}\right)^2$ increases. Thus, the value of the radicand, $r^2 - \left(\frac{d}{2}\right)^2$, will also decrease. As the radicand decreases, and assuming that it remains nonnegative, $\sqrt{r^2 - \left(\frac{d}{2}\right)^2}$ decreases. Therefore, the difference found by $r - \sqrt{r^2 - \left(\frac{d}{2}\right)^2}$ will increase, creating greater values for the depth S.

81. **CHALLENGE** Show that $\sqrt{\frac{8^n \cdot 2^7}{4^{-n}}} = 2^{2n+3}\sqrt{2^{n+1}}$.

SOLUTION:

$$\begin{aligned}\sqrt{\frac{8^n \cdot 2^7}{4^{-n}}} &= \sqrt{\frac{\left(2^3\right)^n \cdot 2^7}{\left(2^2\right)^{-n}}}\\ &= \sqrt{\frac{2^{3n} \cdot 2^7}{2^{-2n}}}\\ &= \sqrt{2^{(3n+7)-(-2n)}}\\ &= \sqrt{2^{5n+7}}\\ &= \sqrt{2^{4n+6} \cdot 2^{n+1}}\\ &= \sqrt{2^{4n+6}} \cdot \sqrt{2^{n+1}}\\ &= 2^{2n+3}\sqrt{2^{n+1}}\end{aligned}$$

83. **PREWRITE** Your Senior Project is to tutor an underclassman for four tutoring sessions on power and radical functions. Make a plan for writing that addresses purpose, audience, a controlling idea, logical sequence, and time frame for completion.

SOLUTION:

Sample answer:
Purpose: To ultimately help an underclassman better understand what power functions are and how they are related to radical functions.
Audience: Either an Algebra 1 or Algebra 2 student.
Controlling idea: Understand the relationship between power and radical functions and how they can be used to solve equations.
Logical sequence and timeframe:
1st session: Practice recognizing power functions.
2nd session: Practice graphing power functions.
3rd session: Practice writing and graphing exponential functions in radical form.
4th session: Practice solving equations involving radicals.

85. **REASONING** Consider $f(x) = x^{\frac{1}{n}} + 5$. How would you expect the graph of the function to change as n increases if n is odd and greater than or equal to 3?

SOLUTION:

Sample answer: As n increases, the value of $\frac{1}{n}$ approaches 0. This means that the value of $x^{\frac{1}{n}}$ will approach 1 when x is positive and −1 when x is negative. Therefore, for positive values of $x, f(x)$ will approach 1 + 5 or 6 and will resemble the line y = 6. For negative values of $x, f(x)$ will approach −1 + 5 or 4 and will resemble the line y = 4.

87. FINANCE If you deposit \$1000 in a savings account with an interest rate of r compounded annually, then the balance in the account after 3 years is given by $B(r) = 1000(1 + r)^3$, where r is written as a decimal.
a. Find a formula for the interest rate r required to achieve a balance of B in the account after 3 years.
b. What interest rate will yield a balance of \$1100 after 3 years?

SOLUTION:

a. Solve $B(r) = 1000(1 + r)^3$ for r. Substitute B for $B(r)$.

$$B(r) = 1000(1 + r)^3$$
$$B = 1000(1 + r)^3$$
$$\frac{B}{1000} = (1 + r)^3$$
$$\sqrt[3]{\frac{B}{1000}} = 1 + r$$
$$\frac{\sqrt[3]{B}}{10} = 1 + r$$
$$\frac{\sqrt[3]{B}}{10} - 1 = r$$

The formula for the interest rate r required is $r = -1 + \frac{\sqrt[3]{B}}{10}$.

b. Substitute $B = 1100$ into the equation found in part **a.**

$$r = -1 + \frac{\sqrt[3]{B}}{10}$$
$$r = -1 + \frac{\sqrt[3]{1100}}{10}$$
$$r = 0.0323$$

The interest rate needed is 3.23%.

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. State the domain of each new function.

89. $f(x) = \frac{x}{x+1}$

$g(x) = x^2 - 1$

SOLUTION:

$$\begin{aligned}(f+g)(x) &= \left(\frac{x}{x+1}\right) + (x^2-1)\\ &= \frac{x}{x+1} + (x^2-1)\cdot\frac{(x+1)}{(x+1)}\\ &= \frac{x}{x+1} + \frac{x^3+x^2-x-1}{x+1}\\ &= \frac{x^3+x^2-1}{x+1}\end{aligned}$$

The denominator cannot equal zero. So, $D = (-\infty, -1) \cup (-1, \infty)$.

$$\begin{aligned}(f-g)(x) &= \left(\frac{x}{x+1}\right) - (x^2-1)\\ &= \frac{x}{x+1} - (x^2-1)\cdot\frac{(x+1)}{(x+1)}\\ &= \frac{x}{x+1} - \frac{x^3+x^2-x-1}{x+1}\\ &= \frac{-x^3-x^2+2x+1}{x+1}\end{aligned}$$

The denominator cannot equal zero. So, $D = (-\infty, -1) \cup (-1, \infty)$.

$$\begin{aligned}(f\cdot g)(x) &= \left(\frac{x}{x+1}\right)\cdot(x^2-1)\\ &= \frac{x}{x+1}\cdot\frac{x^2-1}{1}\\ &= \frac{x}{x+1}\cdot\frac{(x+1)(x-1)}{1}\\ &= \frac{x(x-1)}{1}\\ &= x^2 - x\end{aligned}$$

The denominator of $f(x)$ cannot equal zero. So, $D = (-\infty, -1) \cup (-1, \infty)$.

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{\left(\frac{x}{x+1}\right)}{(x^2-1)}\\ &= \frac{x}{x+1} \div (x^2-1)\\ &= \frac{x}{x+1}\cdot\frac{1}{x^2-1}\\ &= \frac{x}{x^3+x^2-x-1}\end{aligned}$$

The denominator will equal zero when $x = \pm 1$. So, $D = (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

Use the graph of $f(x)$ to graph $g(x) = |f(x)|$ and $h(x) = f(|x|)$.

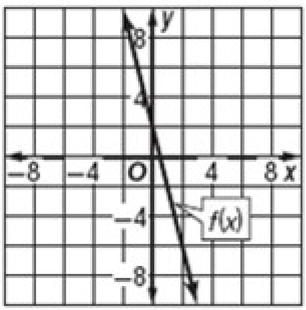
91. $f(x) = -4x + 2$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−3	−2	−1	0	1	2	3
$f(x)$	14	10	6	2	−2	−6	−10

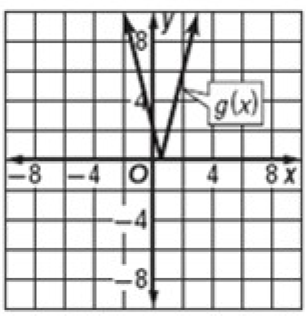
Use these points to construct a graph.



$g(x)$ is the graph of the absolute values of $f(x)$. Evaluate $g(x)$ for several x-values in its domain.

x	−3	−2	−1	0	1	2	3
$g(x)$	14	10	6	2	2	6	10

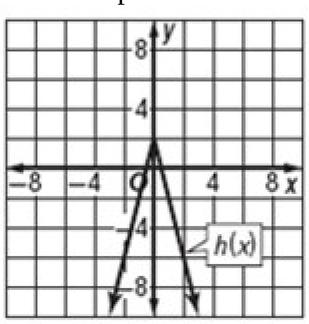
Use these points to construct a graph.



$h(x)$ is the graph of $f(x)$ for the absolute values of x. Evaluate $h(x)$ for several x-values in its domain.

x	−3	−2	−1	0	1	2	3
$h(x)$	−10	−6	−2	2	−2	−6	−10

Use these points to construct a graph.



2-1 Power and Radical Functions

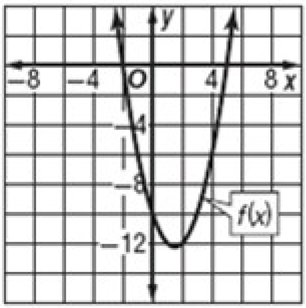
93. $f(x) = x^2 - 3x - 10$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−3	−2	0	1	2	5	6
$f(x)$	8	0	−10	−12	−12	0	8

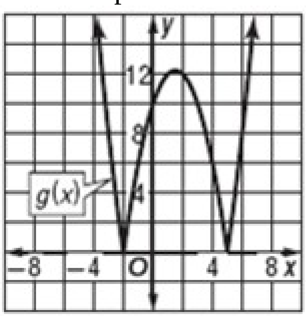
Use these points to construct a graph.



$g(x)$ is the graph of the absolute values of $f(x)$. Evaluate $g(x)$ for several x-values in its domain.

x	−3	−2	0	1	2	5	6
$g(x)$	8	0	10	12	12	0	8

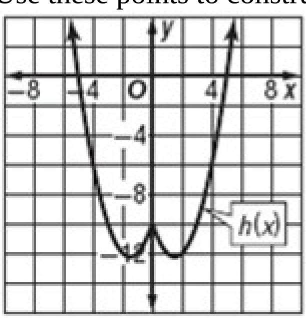
Use these points to construct a graph.



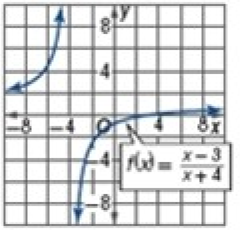
$h(x)$ is the graph of $f(x)$ for the absolute values of x. Evaluate $h(x)$ for several x-values in its domain.

x	−3	−2	−1	0	1	2	3
$h(x)$	−10	−12	−12	−10	−12	−12	−10

Use these points to construct a graph.



Use the graph of each function to estimate intervals to the nearest 0.5 unit on which the function is increasing, decreasing, or constant. Support the answer numerically.

95. 

SOLUTION:

From the graph, we can estimate that f is increasing on $(-\infty, -4)$ and increasing on $(-4, \infty)$. Create a table of values using x-values in each interval.

$(-\infty, -4)$

x	−12	−10	−8	−6	−4.1
***f*(*x*)**	1.86	2.17	2.75	4.5	71

$(-4, \infty)$

x	−3.9	−2	0	2	4
***f*(*x*)**	−69	−2.5	−0.75	−0.17	0.13

The tables support that f is increasing on $(-\infty, -4)$ and increasing on $(-4, \infty)$.

Simplify.

97. $\dfrac{\frac{1}{2} + \sqrt{3}i}{1 - \sqrt{2}i}$

SOLUTION:

$$\begin{aligned}
\frac{\frac{1}{2}+\sqrt{3}i}{1-\sqrt{2}i} &= \frac{\frac{1}{2}+\sqrt{3}i}{1-\sqrt{2}i}\cdot\frac{1+\sqrt{2}i}{1+\sqrt{2}i}\\
&= \frac{\frac{1}{2}+\frac{\sqrt{2}}{2}i+\sqrt{3}i+\sqrt{6}i^2}{1-2i^2}\\
&= \frac{\frac{1}{2}+\frac{\sqrt{2}}{2}i+\sqrt{3}i-\sqrt{6}}{3}\\
&= \frac{1}{6}+\frac{\sqrt{2}}{6}i+\frac{\sqrt{3}i}{3}-\frac{\sqrt{6}}{3}\\
&= \left(\frac{1}{6}-\frac{\sqrt{6}}{3}\right)+\left(\frac{\sqrt{3}}{3}+\frac{\sqrt{2}}{6}\right)i
\end{aligned}$$

99. $\frac{(1+i)^2}{(-3+2i)^2}$

SOLUTION:

$$\begin{aligned}\frac{(1+i)^2}{(-3+2i)^2} &= \frac{1+2i+i^2}{9-12i+4i^2}\\ &= \frac{1+2i-1}{9-12i-4}\\ &= \frac{2i}{5-12i}\\ &= \frac{2i}{5-12i}\cdot\frac{5+12i}{5+12i}\\ &= \frac{10i+24i^2}{25-144i^2}\\ &= \frac{10i-24}{25+144}\\ &= \frac{10i-24}{169}\\ &= -\frac{24}{169}+\frac{10}{169}i\end{aligned}$$

101. **REVIEW** If $f(x, y) = x^2y^3$ and $f(a, b) = 10$, what is the value of $f(2a, 2b)$?

F 50
G 100
H 160
J 320

SOLUTION:

$f(a,b) = a^2b^3$. So, $a^2b^3 = 10$. Evaluate $f(2a, 2b)$.

$$\begin{aligned}f(2a,2b) &= (2a)^2(2b)^3\\ &= 4a^2\cdot 8b^3\\ &= 32a^2b^3\end{aligned}$$

Substitute $a^2b^3 = 10$.

$$\begin{aligned}f(2a,2b) &= 32a^2b^3\\ &= 32\cdot 10\\ &= 320\end{aligned}$$

The correct answer is J.

103. If $\sqrt[3]{5m+2}=3$, then m = ?

F 3
G 4
H 5
J 6

SOLUTION:

$$\begin{aligned}\sqrt[3]{5m+2}&=3\\5m+2&=27\\5m&=25\\m&=5\end{aligned}$$

Since the each side of the equation was raised to a power, check the solution in the original equation.

$$\begin{aligned}\sqrt[3]{5m+2}&=3\\\sqrt[3]{5(5)+2}&=3\\3&=3\end{aligned}$$

The correct answer is H.