3-1 Exponential Functions

Sketch and analyze the graph of each function. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

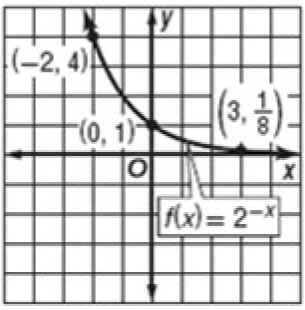
1. $f(x) = 2^{-x}$

SOLUTION:

Evaluate the function for several *x*-values in its domain.

x	−4	−2	−1	0	1	2	4
y	16	4	2	1	0.5	0.25	0.625

Then use a smooth curve to connect each of these ordered pairs.



List the domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

D = $(-\infty, \infty)$; R = $(0, \infty)$; intercept: (0, 1); asymptote: *x*-axis; $\lim_{x \to -\infty} f(x) = \infty, \lim_{x \to \infty} f(x) = 0;$ decreasing on $(-\infty, \infty)$.

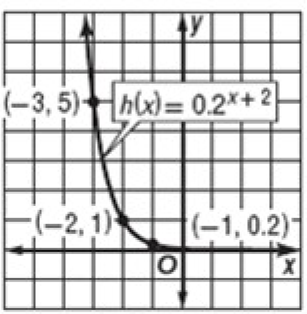
3. $h(x) = 0.2^{x+2}$

SOLUTION:

Evaluate the function for several *x*-values in its domain.

x	−4	−3	−2	0	1	2
y	25	5	1	0.04	0.008	0.0016

Then use a smooth curve to connect each of these ordered pairs.



List the domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

D = $(-\infty, \infty)$; R = $(0, \infty)$; intercept: (0, 0.04); asymptote: *x*-axis; $\lim_{x \to -\infty} h(x) = \infty, \lim_{x \to \infty} h(x) = 0;$ decreasing on $(-\infty, \infty)$

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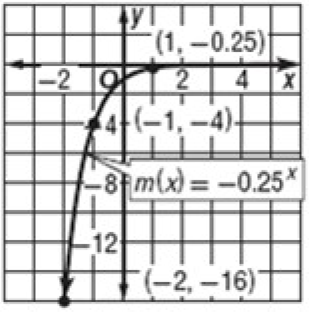
5. $m(x) = -(0.25)^x$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−4	−3	−2	−1	0	1	2
y	–256	−64	−16	−4	−1	−0.25	−0.625

Then use a smooth curve to connect each of these ordered pairs.



List the domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

D = $(-\infty, \infty)$; R = $(-\infty, 0)$; intercept: (0, −1); asymptote: x-axis; $\lim_{x\to-\infty} m(x) = -\infty, \lim_{x\to\infty} m(x) = 0;$ increasing on $(-\infty, \infty)$

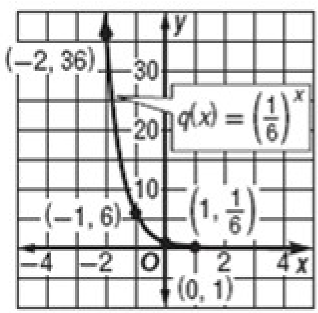
7. $q(x) = \left(\frac{1}{6}\right)^x$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−3	−2	−1	0	1	2
y	216	36	6	1	0.167	0.278

Then use a smooth curve to connect each of these ordered pairs.



List the domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

D = $(-\infty, \infty)$; R = $(0, \infty)$; intercept: (0, 1); asymptote: x-axis; $\lim_{x\to-\infty} q(x) = \infty, \lim_{x\to\infty} q(x) = 0;$ decreasing on $(-\infty, \infty)$

3-1 Exponential Functions

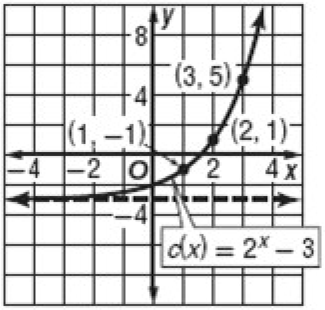
9. $c(x) = 2^x - 3$

SOLUTION:

Evaluate the function for several x-values in its domain.

x	−2	−1	0	1	2	3
y	−2.75	−2.5	−2	−1	1	5

Then use a smooth curve to connect each of these ordered pairs.



D = $(-\infty, \infty)$; R = $(-3, \infty)$; intercept: (0, −2); asymptote: $y = -3$; $\lim_{x \to -\infty} c(x) = -3$, $\lim_{x \to \infty} c(x) = \infty$; increasing on $(-\infty, \infty)$

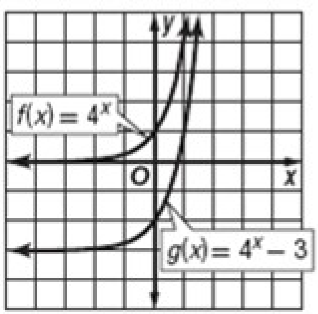
Use the graph of $f(x)$ to describe the transformation that results in the graph of $g(x)$. Then sketch the graphs of $f(x)$ and $g(x)$.

11. $f(x) = 4^x$; $g(x) = 4^x - 3$

SOLUTION:

This function is of the form $f(x) = 4^x$. Therefore, the graph of $g(x)$ is the graph of $f(x)$ translated 3 units down. This is indicated by the subtraction of 3.

Use the graphs of the functions to confirm this transformation.



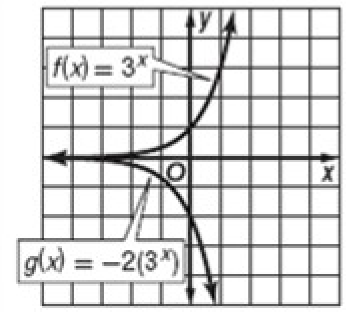
3-1 Exponential Functions

13. $f(x) = 3^x$; $g(x) = -2(3^x)$

SOLUTION:

This function is of the form $f(x) = 3^x$. Therefore, the graph of $g(x)$ is the graph of $f(x)$ reflected across the x-axis and expanded vertically by a factor of 2. These are indicated by the negative sign and the 2 in front of the 3^x.

Use the graphs of the functions to confirm this transformation.

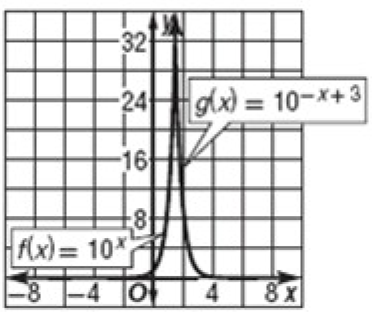


15. $f(x) = 10^x$; $g(x) = 10^{-x+3}$

SOLUTION:

This function is of the form $f(x) = 10^x$. Therefore, the graph of $g(x)$ is the graph of $f(x)$ reflected in the y-axis and translated 3 units to the right. These are indicated by the $-x + 3$ in the exponent.

Use the graphs of the functions to confirm this transformation.

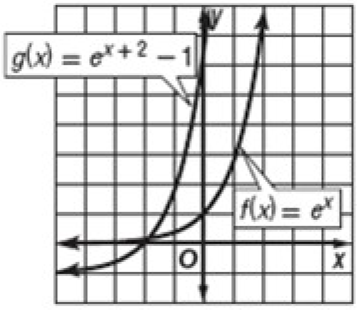


17. $f(x) = e^x$; $g(x) = e^{x+2} - 1$

SOLUTION:

This function is of the form $f(x) = e^x$. Therefore, the graph of $g(x)$ is the graph of $f(x)$ translated 2 units to the left and 1 unit down. These are indicated by the $x + 2$ in the exponent and the subtraction of 1.

Use the graphs of the functions to confirm this transformation.

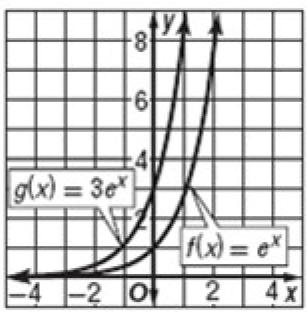


19. $f(x) = e^x$; $g(x) = 3e^x$

SOLUTION:

This function is of the form $f(x) = e^x$. Therefore, the graph of $g(x)$ is the graph of $f(x)$ expanded vertically by a factor of 3. This is indicated by the coefficient of 3.

Use the graphs of the functions to confirm this transformation.



3-1 Exponential Functions

FINANCIAL LITERACY Copy and complete the table below to find the value of an investment A for the given principal P, rate r, and time t if the interest is compounded n times annually.

n	1	4	12	365	continuously
A					

21. P = \$500, r = 3%, t = 5 years

SOLUTION:

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$= 500\left(1+\frac{0.03}{1}\right)^{5}$$
$$\approx \$579.64$$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$= 500\left(1+\frac{0.03}{4}\right)^{4(5)}$$
$$\approx \$580.59$$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$= 500\left(1+\frac{0.03}{12}\right)^{12(5)}$$
$$\approx \$580.81$$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$= 500\left(1+\frac{0.03}{365}\right)^{365(5)}$$
$$\approx \$580.91$$

$$A = Pe^{rt}$$
$$= 500e^{0.03(5)}$$
$$\approx \$580.92$$

n	1	4	12	365	continuously
A	\$579.64	\$580.59	\$580.81	\$580.91	\$580.92

23. P = \$1000, r = 5%, t = 20 years

SOLUTION:

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$= 1000\left(1+\frac{0.05}{1}\right)^{1(20)}$$
$$\approx \$2653.30$$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$= 1000\left(1+\frac{0.05}{4}\right)^{4(20)}$$
$$\approx \$2701.48$$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$= 1000\left(1+\frac{0.05}{12}\right)^{12(20)}$$
$$\approx \$2712.64$$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$
$$= 1000\left(1+\frac{0.05}{365}\right)^{365(20)}$$
$$\approx \$2718.10$$

$$A = Pe^{rt}$$
$$= 1000e^{0.05(20)}$$
$$\approx \$2718.28$$

n	1	4	12	365	continuously
A	\$2653.30	\$2701.48	\$2712.64	\$2718.10	\$2718.28

25. **FINANCIAL LITERACY** Brady acquired an inheritance of $20,000 at age 8, but he will not have access to it until he turns 18.

a. If Brady's inheritance is placed in a savings account earning 4.6% interest compounded monthly, how much will his inheritance be worth on his 18th birthday?

b. How much will Brady's inheritance be worth if it is placed in an account earning 4.2% interest compounded continuously?

SOLUTION:

a. The inheritance is $20,000. The interest rate *r* is 4.6% or 0.046. The interest is compounded monthly, so *n* = 12. There are 10 years from when Brady is 8 to when he is 18. Therefore, *t* = 10. Use the compound interest formula.

$$\begin{aligned} A &= P\left(1+\frac{r}{n}\right)^{nt} \\ &= 20{,}000\left(1+\frac{0.046}{12}\right)^{12(10)} \\ &\approx \$31{,}653.63 \end{aligned}$$

Brady will have $31,653.63 when he turns 18.

b. Use the continuous compound interest formula. The interest rate *r* is 4.6% or 0.046. The time *t* is 10.

$$\begin{aligned} A &= Pe^{rt} \\ &= 20{,}000e^{0.042(10)} \\ &\approx \$30{,}439.23 \end{aligned}$$

Brady will have $30,439.23 when he turns 18.

3-1 Exponential Functions

POPULATION Copy and complete the table to find the population N of an endangered species after a time t given its initial population N_0 and annual rate r or continuous rate k of increase or decline.

t	5	10	15	20	50
N					

27. N_0 = 15,831, r = −4.2%

SOLUTION:

The initial amount N_0 is 15,831 and the annual rate of decline is r = −4.2%. Use the exponential decay formula.

$$N = N_0(1+r)^t$$
$$= 15,831(1-0.042)^5$$
$$\approx 12,774$$

$$N = N_0(1+r)^t$$
$$= 15,831(1-0.042)^{10}$$
$$\approx 10,308$$

$$N = N_0(1+r)^t$$
$$= 15,831(1-0.042)^{15}$$
$$\approx 8317$$

$$N = N_0(1+r)^t$$
$$= 15,831(1-0.042)^{20}$$
$$\approx 6711$$

$$N = N_0(1+r)^t$$
$$= 15,831(1-0.042)^{50}$$
$$\approx 1853$$

t	5	10	15	20	50
N	12,774	10,308	8317	6711	1853

29. $N_0 = 17{,}692$, $k = 2.02\%$

SOLUTION:

The initial amount N_0 is 17,692 and the continuous rate of increase is k =2.02%. Use the continuous exponential growth formula.

$$\begin{aligned} N &= N_0e^{kt} \\ &= 17{,}692e^{0.0202(5)} \\ &\approx 19{,}572 \end{aligned}$$

$$\begin{aligned} N &= N_0e^{kt} \\ &= 17{,}692e^{0.0202(10)} \\ &\approx 21{,}652 \end{aligned}$$

$$\begin{aligned} N &= N_0e^{kt} \\ &= 17{,}692e^{0.0202(15)} \\ &\approx 23{,}953 \end{aligned}$$

$$\begin{aligned} N &= N_0e^{kt} \\ &= 17{,}692e^{0.0202(20)} \\ &\approx 26{,}499 \end{aligned}$$

$$\begin{aligned} N &= N_0e^{kt} \\ &= 17{,}692e^{0.0202(50)} \\ &\approx 48{,}575 \end{aligned}$$

t	5	10	15	20	50
N	19,572	21,652	23,953	26,499	48,575