Perform the indicated operations. If the matrix does not exist, write *impossible*.

31. $3\left(\begin{bmatrix} -2 & 0 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ -3 & -4 \end{bmatrix}\right)$

SOLUTION:

$$3\left(\begin{bmatrix} -2 & 0 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ -3 & -4 \end{bmatrix}\right) = 3\begin{bmatrix} -1 & 9 \\ 3 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} -3 & 27 \\ 9 & 12 \end{bmatrix}$$

32. $\begin{bmatrix} 2 \\ -6 \end{bmatrix} - \begin{bmatrix} -3 \\ 2 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \end{bmatrix}$

SOLUTION:

$$\begin{bmatrix} 2 \\ -6 \end{bmatrix} - \begin{bmatrix} -3 \\ 2 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 11 \\ -8 \end{bmatrix}$$

33. **RETAIL** Current Fashions buys shirts, jeans and shoes from a manufacturer, marks them up, and then sells them. The table shows the purchase price and the selling price.

Item	Purchase Price	Selling Price
shirts	\$15	\$35
jeans	\$25	\$55
shoes	\$30	\$85

a. Write a matrix for the purchase price.
b. Write a matrix for the selling price.
c. Use matrix operations to find the profit on 1 shirt, 1 pair of jeans, and 1 pair of shoes.

SOLUTION:

a. Purchase price:

$$\begin{bmatrix} 15 \\ 25 \\ 30 \end{bmatrix}$$

b. Selling price:

$$\begin{bmatrix} 35 \\ 55 \\ 85 \end{bmatrix}$$

c. Subtract the matrices.

$$\begin{bmatrix} 35 \\ 55 \\ 85 \end{bmatrix} - \begin{bmatrix} 15 \\ 25 \\ 30 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \\ 55 \end{bmatrix}$$

Find each product, if possible.

34. $\begin{bmatrix} 3 & -7 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ -5 \end{bmatrix}$

SOLUTION:

$$\begin{bmatrix} 3 & -7 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ -5 \end{bmatrix} = \begin{bmatrix} 62 \end{bmatrix}$$

35. $\begin{bmatrix} -3 & 0 & 2 \\ 6 & -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 8 & -1 \\ -4 & 3 \\ 6 & 7 \end{bmatrix}$

SOLUTION:

$$\begin{bmatrix} -3 & 0 & 2 \\ 6 & -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 8 & -1 \\ -4 & 3 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} -12 & 17 \\ 82 & 26 \end{bmatrix}$$

36. $\begin{bmatrix} 2 & 11 \\ 0 & -3 \\ -6 & 7 \end{bmatrix} \cdot \begin{bmatrix} 0 & 8 & -5 \\ 12 & 0 & 9 \\ 4 & -6 & 7 \end{bmatrix}$

SOLUTION:

The inner dimensions of the matrices are not equal. So, the matrices cannot be multiplied.

37. **GROCERIES** Martin bought 1 gallon of milk, 2 apples, 4 frozen dinners, and 1 box of cereal. The following matrix shows the prices for each item respectively.
[$2.59 $0.49 $5.25 $3.99]
Use matrix multiplication to find the total amount of money Martin spent at the grocery store.

SOLUTION:

$$\begin{bmatrix} 2.59 & 0.49 & 5.25 & 3.99 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 28.56 \end{bmatrix}$$

So, Martin spent $28.56.

Evaluate each determinant.

38. $\begin{vmatrix} 2 & 4 \\ 7 & -3 \end{vmatrix}$

SOLUTION:

$$\begin{vmatrix} 2 & 4 \\ 7 & -3 \end{vmatrix} = -6 - 28$$
$$= -34$$

39. $\begin{vmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{vmatrix}$

SOLUTION:

$$\begin{vmatrix} 2 & 3 & -1 \\ 0 & 2 & 4 \\ -2 & 5 & 6 \end{vmatrix} = 2(12-20) - 3(0+8) - 1(0+4)$$
$$= -16 - 24 - 4$$
$$= -44$$

Use Cramer's Rule to solve each system of equations.

40. $3x - y = 0$
$5x + 2y = 22$

SOLUTION:

Let $C = \begin{bmatrix} 3 & -1 \\ 5 & 2 \end{bmatrix}$.

$$|C| = \begin{vmatrix} 3 & -1 \\ 5 & 2 \end{vmatrix}$$
$$= 11$$

$$x = \frac{\begin{vmatrix} 0 & -1 \\ 22 & 2 \end{vmatrix}}{11}$$
$$= \frac{22}{11}$$
$$= 2$$

$$y = \frac{\begin{vmatrix} 3 & 0 \\ 5 & 22 \end{vmatrix}}{11}$$
$$= \frac{66}{11}$$
$$= 6$$

Therefore, the solution is (2, 6).

41. $5x + 2y = 4$
$3x + 4y + 2z = 6$
$7x + 3y + 4z = 29$

SOLUTION:

Let $C = \begin{bmatrix} 5 & 2 & 0 \\ 3 & 4 & 2 \\ 7 & 3 & 4 \end{bmatrix}$.

$$\begin{aligned} |C| &= 5(16-6) - 2(12-14) + 0(9-28) \\ &= 50 + 4 + 0 \\ &= 54 \end{aligned}$$

$$\begin{aligned} x &= \frac{\begin{vmatrix} 4 & 2 & 0 \\ 6 & 4 & 2 \\ 29 & 3 & 4 \end{vmatrix}}{54} \\ &= \frac{108}{54} \\ &= 2 \end{aligned}$$

$$\begin{aligned} y &= \frac{\begin{vmatrix} 5 & 4 & 0 \\ 3 & 6 & 2 \\ 7 & 29 & 4 \end{vmatrix}}{54} \\ &= -\frac{162}{54} \\ &= -3 \end{aligned}$$

$$\begin{aligned} z &= \frac{\begin{vmatrix} 5 & 2 & 4 \\ 3 & 4 & 6 \\ 7 & 3 & 29 \end{vmatrix}}{54} \\ &= \frac{324}{54} \\ &= 6 \end{aligned}$$

The solution is (2, −3, 6).

42. **JEWELRY** Alana paid \$98.25 for 3 necklaces and 2 pairs of earrings. Petra paid \$133.50 for 2 necklaces and 4 pairs of earrings. Use Cramer's Rule to find out how much 1 necklace costs and how much 1 pair of earrings costs.

SOLUTION:

Let *x* be the number of necklaces and *y* be the number of pairs of earrings.

$$3x + 2y = 98.25$$
$$2x + 4y = 133.50$$

Let $C = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$.

$$\begin{aligned} |C| &= \begin{vmatrix} 3 & 2 \\ 2 & 4 \end{vmatrix} \\ &= 12 - 4 \\ &= 8 \end{aligned}$$

$$\begin{aligned} x &= \frac{\begin{vmatrix} 98.25 & 2 \\ 133.50 & 4 \end{vmatrix}}{8} \\ &= \frac{126}{8} \\ &= 15.75 \end{aligned}$$

$$\begin{aligned} y &= \frac{\begin{vmatrix} 3 & 98.25 \\ 2 & 133.50 \end{vmatrix}}{8} \\ &= \frac{204}{8} \\ &= 25.50 \end{aligned}$$

So, the cost of 1 necklace is \$15.75 and a pair of earrings is \$25.50.

Find the inverse of each matrix, if it exists.

43. $\begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix}$

SOLUTION:

$$\begin{vmatrix} 7 & 4 \\ 3 & 2 \end{vmatrix} = 7(2) - 3(4)$$
$$= 14 - 12$$
$$= 2$$

Since the determinant is non-zero, the inverse exists.

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & -4 \\ -3 & 7 \end{bmatrix}$$

44. $\begin{bmatrix} 2 & 5 \\ -5 & -13 \end{bmatrix}$

SOLUTION:

$$\begin{vmatrix} 2 & 5 \\ -5 & -13 \end{vmatrix} = -26 + 25$$
$$= -1$$

Since the determinant is non-zero, the inverse exists.

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} -13 & -5 \\ 5 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 13 & 5 \\ -5 & -2 \end{bmatrix}$$

45. $\begin{bmatrix} 6 & -3 \\ -8 & 4 \end{bmatrix}$

SOLUTION:

$$\begin{vmatrix} 6 & -3 \\ -8 & 4 \end{vmatrix} = 24 - 24$$
$$= 0$$

Since the determinant is 0, the inverse does not exist.

Use a matrix equation to solve each system of equations.

46. $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

SOLUTION:

Let $A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$.

$$A^{-1} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -12 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -12 \end{bmatrix}$$

The solution of the system is (8, −12).

47. $\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

SOLUTION:

Let $A = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$.

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$$

$$\frac{1}{7}\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 14 \\ 7 \end{bmatrix}$$
$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

The solution of the system is (2, 1).

48. **HEALTH FOOD** Heath sells nuts and raisins by the pound. Sonia bought 2 pounds of nuts and 2 pounds of raisins for $23.50. Drew bought 3 pounds of nuts and 1 pound of raisins for $22.25. What is the cost of 1 pound of nuts and 1 pound of raisins?

SOLUTION:

Let x = cost of 1 pound of nuts and y = cost of 1 pound of raisins.

$$2x + 2y = 23.50$$
$$3x + y = 22.25$$

The matrix equation is $\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 23.50 \\ 22.25 \end{bmatrix}$.

The inverse of $\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$ is $-\frac{1}{4}\begin{bmatrix} 1 & -2 \\ -3 & 2 \end{bmatrix}$.

$$-\frac{1}{4}\begin{bmatrix} 1 & -2 \\ -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} 1 & -2 \\ -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 23.50 \\ 22.25 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} -21 \\ -26 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5.25 \\ 6.50 \end{bmatrix}$$

The cost of 1 pound of nuts is $5.25 and 1 pound of raisins is $6.50.