5-4 Analyzing Graphs of Polynomial Functions

Graph each polynomial equation by making a table of values.

2. \( f(x) = -2x^4 + 4x^3 + 2x^2 + x - 3 \)

**SOLUTION:**

Make a table of values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-61</td>
</tr>
<tr>
<td>-1</td>
<td>-8</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

Plot the points on the coordinate plane and connect them by a smooth curve.
5-4 Analyzing Graphs of Polynomial Functions

4. \( f(x) = -4x^4 + 5x^3 + 2x^2 + 3x + 1 \)

**SOLUTION:**
Make a table of values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-101</td>
</tr>
<tr>
<td>-1</td>
<td>-9</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>-9</td>
</tr>
</tbody>
</table>

Plot the points on the coordinate plane and connect them by a smooth curve.
5-4 Analyzing Graphs of Polynomial Functions

Determine the consecutive integer values of \( x \) between which each real zero of each function located. Then draw the graph.

6. \( f(x) = -x^4 + x^3 + 2x^2 + x + 1 \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-17</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>-32</td>
</tr>
</tbody>
</table>

The changes in sign indicate that there are zeros between 2 and 3 and at -1.
5-4 Analyzing Graphs of Polynomial Functions

8. \( f(x) = 2x^4 - x^3 - 3x^2 + 2x - 4 \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>20</td>
</tr>
<tr>
<td>-1</td>
<td>-6</td>
</tr>
<tr>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>1</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>

The changes in sign indicate that there are zeros between -2 and -1 and between 1 and 2.
Graph each polynomial function. Estimate the \( x \)-coordinates at which the relative maxima and relative minima occur. State the domain and range for each function.

10. \( f(x) = 3x^3 - 6x^2 - 2x + 2 \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-127</td>
</tr>
<tr>
<td>-2</td>
<td>-42</td>
</tr>
<tr>
<td>-1</td>
<td>-5</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>23</td>
</tr>
</tbody>
</table>

Graph:

Relative maxima at \( x \approx -0.2 \);
Relative minima at \( x \approx 1.5 \);
Domain: \( \{ \text{All real numbers} \} \)
Range: \( \{ \text{All real numbers} \} \)
12. \( f(x) = -x^3 + 2x^2 - 3x + 4 \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>58</td>
</tr>
<tr>
<td>-2</td>
<td>26</td>
</tr>
<tr>
<td>-1</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>-14</td>
</tr>
</tbody>
</table>

Graph:

No Relative maxima: No Relative minima:
Domain: \{ All real numbers \}
Range: \{ All real numbers \}
5-4 Analyzing Graphs of Polynomial Functions

Complete each of the following.

a. Graph each function by making a table of values.

b. Determine the consecutive integer values of \( x \) between which each real zero is located.

c. Estimate the \( x \)-coordinates at which the relative maxima and minima occur.

14. \( f(x) = x^3 + 3x^2 \)

**SOLUTION:**

\[
\begin{array}{|c|c|}
\hline
x & f(x) \\
\hline
-4 & -16 \\
-3 & 0 \\
-2 & 4 \\
-1 & 2 \\
0 & 0 \\
1 & 4 \\
2 & 20 \\
3 & 54 \\
4 & 112 \\
\hline
\end{array}
\]

b. The changes in sign indicate that there are zeros at \(-3\) and at \(0\).

c. Relative maxima at \( x = -2 \); Relative minima at \( x = 0 \).
16. \( f(x) = x^3 + 4x^2 - 5x \)

**SOLUTION:**

a. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>-42</td>
</tr>
<tr>
<td>-5</td>
<td>0</td>
</tr>
<tr>
<td>-4</td>
<td>20</td>
</tr>
<tr>
<td>-3</td>
<td>24</td>
</tr>
<tr>
<td>-2</td>
<td>18</td>
</tr>
<tr>
<td>-1</td>
<td>8</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>48</td>
</tr>
</tbody>
</table>

b. The changes in sign indicate that there are zeros at -5, 0 and 1.

c. Relative maxima at \( x = -3 \); Relative minima between \( x = 0 \) and \( x = 1 \).
18. \( f(x) = -2x^3 + 12x^2 - 8x \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>22</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>-48</td>
</tr>
<tr>
<td>7</td>
<td>-154</td>
</tr>
</tbody>
</table>

b. The changes in sign indicate that there are zeros at 0, between \( x = 0 \) and \( x = 1 \), and between \( x = 5 \) and \( x = 6 \).

c. Relative maxima: near \( x = 4 \); Relative minima between \( x = 0 \) and \( x = 1 \).
20. \( f(x) = x^4 + 2x - 1 \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>247</td>
</tr>
<tr>
<td>-3</td>
<td>74</td>
</tr>
<tr>
<td>-2</td>
<td>11</td>
</tr>
<tr>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
</tr>
<tr>
<td>3</td>
<td>86</td>
</tr>
<tr>
<td>4</td>
<td>263</td>
</tr>
</tbody>
</table>

b. The changes in sign indicate that there are zeros between \( x = -2 \) and \( x = -1 \) and between \( x = 0 \) and \( x = 1 \).

c. No relative maxima; Relative minima: near \( x = -1 \)
5-4 Analyzing Graphs of Polynomial Functions

22. FINANCIAL LITERACY The average annual price of gasoline can be modeled by the cubic function \( f(x) = 0.0007x^3 - 0.014x^2 + 0.08x + 0.96 \), where \( x \) is the number of years after 1987 and \( f(x) \) is the price in dollars.
   a. Graph the function for \( 0 \leq x \leq 30 \).
   b. Describe the turning points of the graph and its end behavior.
   c. What trends in gasoline prices does the graph suggest?
   d. Is it reasonable that the trend will continue indefinitely? Explain.

**SOLUTION:**

a. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.96</td>
</tr>
<tr>
<td>5</td>
<td>1.0975</td>
</tr>
<tr>
<td>10</td>
<td>1.06</td>
</tr>
<tr>
<td>15</td>
<td>1.3725</td>
</tr>
<tr>
<td>20</td>
<td>2.56</td>
</tr>
<tr>
<td>25</td>
<td>5.1475</td>
</tr>
<tr>
<td>30</td>
<td>9.66</td>
</tr>
</tbody>
</table>

b. Sample answer: The graph has a relative minimum at \( x = 10 \) and then increases as \( x \) increases.

c. The graph suggests a fairly steep continuous increase and gas prices at $5 per gallon by 2012, which could be possible.

d. Sample answer: While it is possible for gasoline prices to continue to soar at this rate, it is likely that alternate forms of transportation and fuel will slow down this rapid increase.
5-4 Analyzing Graphs of Polynomial Functions

Use a graphing calculator to estimate the x-coordinates at which the maxima and minima of each function occur. Round to the nearest hundredth.

24. \( f(x) = -2x^3 + 4x^2 - 5x + 8 \)

**SOLUTION:**
Graph the function.

From the graph, the function has no relative maxima and relative minima.

26. \( f(x) = x^5 - 4x^3 + 3x^2 - 8x - 6 \)

**SOLUTION:**
Graph the function.

From the graph, the relative maxima is at \( x = -1.87 \) and relative minima is at \( x = 1.52 \).
Sketch the graph of polynomial functions with the following characteristics.

28. an even function with zeros at $-2, 1, 3,$ and $5$

**SOLUTION:**
An even-degree function has an even number of real zeros and the end behavior is in the same direction. So draw a graph that crosses the $x$-axis at $-2, 1, 3,$ and $5$.

30. a 5-degree function with zeros at $-4, -1,$ and $3,$ maximum at $x = -2$

**SOLUTION:**
A 5-degree function has 5 zeros and end behavior in opposite directions. Draw a graph with zeros at $-4, -1,$ and $3,$ and a maximum at $x = -2.$
32. An even function with a minimum at \( x = 3 \) and a positive leading coefficient

**SOLUTION:**
An even function with a a positive leading coefficient has end behavior in the same direction. Draw a graph with a minimum at \( x = 3 \) with an even number of zeros.

Complete each of the following.

a. Estimate the \( x \)-coordinate of every turning point and determine if those coordinates are relative maxima or relative minima.

b. Estimate the \( x \)-coordinate of every zero.

c. Determine the smallest possible degree of the function.

d. Determine the domain and range of the function.

**SOLUTION:**

a. Relative maxima: \( x = -3.5 \) and \( x = -1 \); Relative minima: \( x = -2.5 \) and \( x = 2 \)

b. The zeros are at: \( x = -1.75, -0.25 \) and \( 3.5 \).

c. Since the graph has 4 turning points, the smallest possible degree of the polynomial function is \( (4 + 1) \) or 5.

d. Domain: \{all real numbers\}; Range: \{all real numbers\};
5-4 Analyzing Graphs of Polynomial Functions

**SOLUTION:**

a. Relative maxima: $x = -2$ and $x = 2.5$; Relative minima: $x = 1$

b. The zeros are at: $x = -3.5$, and $x = -0.5$.

c. Since the graph has 3 turning points, the smallest possible degree of the polynomial function is $(3 + 1)$ or 4.

d. Domain: $\{\text{all real numbers}\}$; Range: $\{y \mid y \leq 4.1\}$

**SOLUTION:**

a. Relative maxima: $x = -3.5$, $x = -1.75$ and $x = 1$; Relative minima: $x = -2.5$, $x = -1$ and $x = 2$

b. The zeros are at: $x = -4$, $-3$, $0$, $1.5$, and $2.75$.

c. Since the graph has 6 turning points, the smallest possible degree of the polynomial function is $(6 + 1)$ or 7.

d. Domain: $\{\text{all real numbers}\}$; Range: $\{\text{all real numbers}\}$;
40. CCSS REASONING The number of subscribers using pagers in the United States can be modeled by

\[ f(x) = 0.015x^4 - 0.44x^3 + 3.46x^2 - 2.7x + 9.68 \]

where \( x \) is the number of years after 1990 and \( f(x) \) is the number of subscribers in millions.

a. Graph the function.
b. Describe the end behavior of the graph.
c. What does the end behavior suggest about the number of pager subscribers?
d. Will this trend continue indefinitely? Explain your reasoning.

**SOLUTION:**

a. Use a graphing calculator to graph the function.

![Graph of function](image)

b. As \( x \) increases, \( f(x) \) increases.
c. Sample answer: The graph suggests that the number of pager subscribers will increase dramatically and continue to increase.
d. Sample answer: The graph is unreasonable for \( x \geq 15 \) since pager use is currently decreasing rapidly and pagers have been replaced by more efficient products.
For each function,

a. determine the zeros, x- and y-intercepts, and turning points,

b. determine the axis of symmetry, and

c. determine the intervals for which it is increasing, decreasing, or constant.

42. \( f(x) = x^4 - 8x^2 + 16 \)

**SOLUTION:**

Graph the function.

---

**a.** The zeros of the function are: \( x = 2, -2 \);

x-intercepts: \( x = 2, -2 \);

y-intercept: \( y = 16 \)

Turning points: \( x = -2, 0, 2 \)

**b.** The axis of symmetry is \( x = 0 \).

**c.** The function is increasing in the intervals \( -2 < x < 0 \) and \( x > 2 \) and decreasing in \( x < -2 \) and \( 0 < x < 2 \).
5-4 Analyzing Graphs of Polynomial Functions

44. \(f(x) = -2x^4 + 4x^3 + 2x^2 + x - 3\)

**SOLUTION:**
Graph the function.

![Graph of the function](image)

a. The zeros of the function are: \(x \approx -1\) and 0.
   x-intercepts: \(x \approx -1\) and 0;
   y-intercept: \(y = 0\);
   Turning point: \(x \approx -0.5\);

b. No axis of symmetry.

c. Increasing: \(x \leq -0.5\)
   Decreasing: \(x \geq -0.5\)

46. **MULTIPLE REPRESENTATIONS** Consider the following function.

\[f(x) = x^4 - 8.65x^3 + 27.34x^2 - 37.2285x + 18.27\]

a. **ANALYTICAL** What are the degree, leading coefficient, and end behavior?

b. **TABULAR** Make a table of integer values \(f(x)\) if \(-4 \leq x \leq 4\) How many zeros does the function appear to have from the table?

c. **GRAPHICAL** Graph the function by using a graphing calculator.

d. **GRAPHICAL** Change the viewing window to \([0, 4]\) scl: 1 by \([-0.4, 0.4]\) scl: 0.2. What conclusions can you make from this new view of the graph?

**SOLUTION:**

a. degree: 4; leading coefficient: 1;
   End behavior: as \(x \to -\infty, f(x) \to +\infty\), as \(x \to +\infty, f(x) \to +\infty\)

b. 

The changes in sign indicate that there are zeros between \( x = 1 \) and \( x = 2 \) and between \( x = 2 \) and \( x = 3 \). So, there are two zeros.

c.

d.

Sample answer: Sometimes it is necessary to have a more accurate viewing window or to change the interval values of the table function in order to assess the graph more accurately.
5-4 Analyzing Graphs of Polynomial Functions

48. REASONING The table below shows the values of \( g(x) \), a cubic function. Could there be a zero between \( x = 2 \) and \( x = 3 \)? Explain your reasoning.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -2 )</th>
<th>( -1 )</th>
<th>( 0 )</th>
<th>( 1 )</th>
<th>( 2 )</th>
<th>( 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td>( 4 )</td>
<td>( -2 )</td>
<td>( -1 )</td>
<td>( 1 )</td>
<td>( -2 )</td>
<td>( -2 )</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Sample answer: No; the cubic function is of degree 3 and cannot have any more than three zeros. Those zeros are located between \(-2\) and \(-1\), 0 and 1, and 1 and 2.

50. CCSS ARGUMENTS Determine whether the following statement is sometimes, always, or never true. Explain your reasoning.

*For any continuous polynomial function, the y-coordinate of a turning point is also either a relative maximum or relative minimum.*

**SOLUTION:**
Sample answer: Always; the definition of a turning point of a graph is a point in which the graph stops increasing and begins to decrease, causing a maximum or stops decreasing and begins to increase, causing a minimum.

52. REASONING A function is said to be odd if for every \( x \) in the domain, \( -f(x) = f(-x) \). Is every odd-degree polynomial function also an odd function? Explain.

**SOLUTION:**
Sample answer: No; \( f(x) = x^3 + 2x^2 \) is an odd degree, but \( -f(1) \neq f(-1) \).

54. Which of the following is the factorization of \( 2x - 15 + x^2 \)?

A. \( (x - 3)(x - 5) \)
B. \( (x - 3)(x + 5) \)
C. \( (x + 3)(x - 5) \)
D. \( (x + 3)(x + 5) \)

**SOLUTION:**
First rewrite the equation with the terms in descending order by degree. Then factor.

\[
x^2 + 2x - 15 = x^2 + 5x - 3x - 15
= x(x + 5) - 3(x + 5)
= (x - 3)(x + 5)
\]

The correct choice is B.
5-4 Analyzing Graphs of Polynomial Functions

56. Which polynomial represents \((4x^2 + 5x - 3)(2x - 7)\)?

\[
\begin{align*}
\text{F} & \quad 8x^3 - 18x^2 - 41x - 21 \\
\text{G} & \quad 8x^3 + 18x^2 + 29x - 21 \\
\text{H} & \quad 8x^3 - 18x^2 - 41x + 21 \\
\text{J} & \quad 8x^3 + 18x^2 - 29x + 21
\end{align*}
\]

**SOLUTION:**

\[
\begin{align*}
(4x^2 + 5x - 3)(2x - 7) &= 8x^3 + 10x^2 - 6x - 28x^2 - 35x + 21 \\
&= 8x^3 - 18x^2 - 41x + 21
\end{align*}
\]

The correct choice is H.

For each graph,

a. describe the end behavior,

b. determine whether it represents an odd-degree or an even-degree function, and

c. state the number of real zeros.

58. **SOLUTION:**

a. \(f(x) \rightarrow +\infty \text{ as } x \rightarrow -\infty.\) \(f(x) \rightarrow +\infty \text{ as } x \rightarrow +\infty.\)

b. Since the end behavior is in the same direction, it is an even-degree function.

c. The graph intersects the x-axis at two points, so there are two real zeros.

60. **SOLUTION:**

a. \(f(x) \rightarrow +\infty \text{ as } x \rightarrow -\infty.\) \(f(x) \rightarrow +\infty \text{ as } x \rightarrow +\infty.\)

b. Since the end behavior is in the same direction, it is an even-degree function.

c. The graph intersects the x-axis at four points, so there are four real zeros.
5-4 Analyzing Graphs of Polynomial Functions

Simplify.
62. \((4y^3 + 18y^2 + 5y - 12) ÷ (y + 4)\)

**SOLUTION:**
Use synthetic division method.

\[
\begin{array}{c|cccc}
-4 & 4 & 18 & 5 & -12 \\
 & & -16 & -8 & 12 \\
--- & --- & --- & --- & ---
\end{array}
\]

\((4y^3 + 18y^2 + 5y - 12) ÷ (y + 4) = 4y^2 + 2y - 3\)

64. **CHEMISTRY** Tanisha needs 200 milliliters of a 48% concentration acid solution. She has 60% and 40% concentration solutions in her lab. How many milliliters of 40% acid solution should be mixed with 60% acid solution to make the required amount of 48% acid solution?

**SOLUTION:**
Let \(x\) be the amount of 60% acid solution. Therefore,

\[
(0.6)x + (0.4)(200 - x) = (0.48)(200).
\]

\[
0.6x + 80 - 0.4x = 96
\]

\[
0.2x = 16
\]

\[
x = 80
\]

60% solution: 80 mL
40% solution: 200 - 80 = 120 mL

Factor.
66. \(y^2 - 5y - 8y + 40\)

**SOLUTION:**
\(y^2 - 5y - 8y + 40 = y(y - 5) - 8(y - 5)\)

\(= (y - 5)(y - 8)\)

68. \(b^2 - 4b - 21\)

**SOLUTION:**
\(b^2 - 4b - 21 = b^2 + 3b - 7b - 21\)

\(= b(b + 3) - 7(b + 3)\)

\(= (b + 3)(b - 7)\)
5-4 Analyzing Graphs of Polynomial Functions

70. \(4x^2 - 7x - 15\)

\textbf{SOLUTION:}

\[
4x^2 - 7x - 15 = 4x^2 - 12x + 5x - 15 \\
= 4x(x - 3) + 5(x - 3) \\
= (x - 3)(4x + 5)
\]