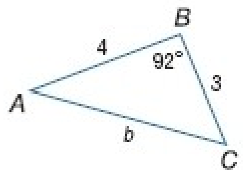
12-5 Law of Cosines

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

1. 

SOLUTION:

Use the Law of Cosines to find the missing side length.

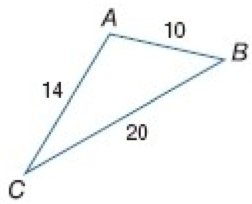
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 3^2 + 4^2 - 2(3)(4)\cos 92^\circ$$
$$b^2 \approx 25.8$$
$$b \approx 5.1$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin A}{3} \approx \frac{\sin 92^\circ}{5.1}$$
$$\sin A \approx \frac{3\sin 92^\circ}{5.1}$$
$$A \approx 36^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180^\circ - (36^\circ + 92^\circ)$$
$$\approx 52^\circ$$

2. 

SOLUTION:

Use the Law of Cosines to find the measure of the largest angle, $\angle A$.

$$
\begin{aligned}
a^2 &= b^2 + c^2 - 2bc\cos A \\
20^2 &= 14^2 + 10^2 - 2(14)(10)\cos A \\
\frac{20^2 - 14^2 - 10^2}{-2(14)(10)} &= \cos A \\
112^\circ &\approx A
\end{aligned}
$$

Use the Law of Sines to find the measure of angle, $\angle B$.

$$
\begin{aligned}
\frac{\sin B}{14} &\approx \frac{\sin 112^\circ}{20} \\
\sin B &\approx \frac{14 \sin 112^\circ}{20} \\
B &\approx 40^\circ
\end{aligned}
$$

Find the measure of $\angle C$.

$$
\begin{aligned}
m\angle C &\approx 180^\circ - (112^\circ + 40^\circ) \\
&\approx 28^\circ
\end{aligned}
$$

3. a = 5, b = 8, c = 12

SOLUTION:

Use the Law of Cosines to find the measure of the largest angle, $\angle C$.

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos C \\ 12^2 &= 5^2 + 8^2 - 2(5)(8)\cos C \\ \frac{12^2 - 5^2 - 8^2}{-2(5)(8)} &= \cos C \\ 133^\circ &\approx C \end{aligned}$$

Use the Law of Sines to find the measure of angle, $\angle B$.

$$\begin{aligned} \frac{\sin B}{8} &\approx \frac{\sin 133^\circ}{12} \\ \sin B &\approx \frac{8\sin 133^\circ}{12} \\ B &\approx 29^\circ \end{aligned}$$

Find the measure of $\angle A$.

$$\begin{aligned} m\angle A &\approx 180^\circ - (133^\circ + 29^\circ) \\ &\approx 18^\circ \end{aligned}$$

4. $B = 110°$, $a = 6$, $c = 3$

SOLUTION:

Use the Law of Cosines to find the missing side length.

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 6^2 + 3^2 - 2(6)(3)\cos 110$$
$$b^2 \approx 57.3$$
$$b \approx 7.6$$

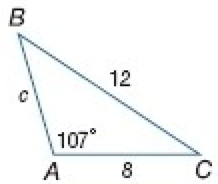
Use the Law of Sines to find a missing angle measure.

$$\frac{\sin A}{6} \approx \frac{\sin 110}{7.6}$$
$$\sin A \approx \frac{6\sin 110}{7.6}$$
$$A \approx 48$$

Find the measure of $\angle C$.

$$m\angle C \approx 180 - (48 + 110)$$
$$\approx 22$$

CCSS PRECISION Determine whether each triangle should be solved by beginning with the Law of *Sines* or the Law of *Cosines*. Then solve the triangle.

5. 

SOLUTION:

Since two sides and an angle opposite one of them of a triangle are given, the triangle should be solved by beginning with the Law of Sines.

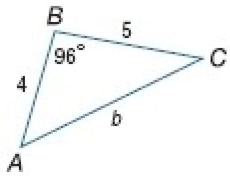
$$\frac{\sin 107^\circ}{12} = \frac{\sin B}{8}$$
$$\sin B = \frac{8 \sin 107^\circ}{12}$$
$$\sin B \approx 0.6375$$
$$B \approx 40^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180^\circ - (107^\circ + 40^\circ)$$
$$\approx 33^\circ$$

Use Law of Sines to find c.

$$\frac{\sin 107^\circ}{12} \approx \frac{\sin 33^\circ}{c}$$
$$c \approx \frac{12 \sin 33^\circ}{\sin 107^\circ}$$
$$c \approx 6.8$$

6. 

SOLUTION:

Since two sides and their included angle of a triangle are given, the triangle should be solved by beginning with the Law of Cosines.

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 5^2 + 4^2 - 2(5)(4)\cos 96°$$
$$b \approx 6.7$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin A}{5} \approx \frac{\sin 96°}{6.7}$$
$$\sin A \approx \frac{5\sin 96°}{6.7}$$
$$A \approx 48°$$

Find the measure of $\angle C$.

$$m\angle C \approx 180° - (48° + 96°)$$
$$\approx 36°$$

7. In ΔRST, $R = 35°$, $s = 16$, and $t = 9$.

SOLUTION:

Since two sides and their included angle of a triangle are given, the triangle should be solved by beginning with the Law of Cosines.

$$r^2 = s^2 + t^2 - 2st\cos R$$
$$r^2 = 16^2 + 9^2 - 2(16)(9)\cos 35°$$
$$r \approx 10.1$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin T}{t} = \frac{\sin R}{r}$$
$$\frac{\sin T}{9} \approx \frac{\sin 35°}{10.1}$$
$$\sin T \approx \frac{9\sin 35°}{10.1}$$
$$T \approx 31°$$

Find the measure of $\angle S$.

$$m\angle S \approx 180° - (35° + 31°) \text{ or } 114°$$

8. **FOOTBALL** In a football game, the quarterback is 20 yards from Receiver A. He turns 40° to see Receiver B, who is 16 yards away. How far apart are the two receivers?

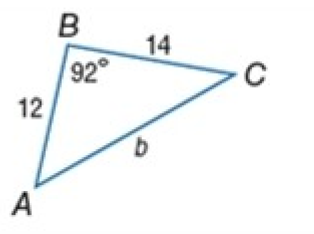
SOLUTION:

Use the Law of Cosines to find the missing side length.

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = 16^2 + 20^2 - 2(16)(20)\cos 40°$$
$$c^2 \approx 165.7$$
$$c \approx 12.9$$

The two receivers are about 12.9 yards apart.

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

10. 

SOLUTION:

Use the Law of Cosines to find the missing side length.

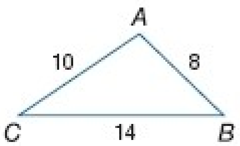
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 14^2 + 12^2 - 2(14)(12)\cos 92^\circ$$
$$b^2 \approx 351.7262$$
$$b \approx 18.75$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin A}{14} \approx \frac{\sin 92^\circ}{18.8}$$
$$\sin A \approx \frac{5\sin 96^\circ}{6.7}$$
$$A \approx 48^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180^\circ - (48^\circ + 92^\circ)$$
$$\approx 40^\circ$$



12.

SOLUTION:

Use the Law of Cosines to find the measure of the largest angle, $\angle A$.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$14^2 = 10^2 + 8^2 - 2(10)(8)\cos A$$
$$\frac{14^2 - 10^2 - 8^2}{-2(10)(8)} = \cos A$$
$$102^\circ \approx A$$

Use the Law of Sines to find the measure of angle, $\angle B$.

$$\frac{\sin 102^\circ}{14} \approx \frac{\sin B}{10}$$
$$\sin B \approx \frac{10\sin 102^\circ}{14}$$
$$B \approx 44^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180^\circ - (102^\circ + 44^\circ)$$
$$\approx 34^\circ$$

14. $C = 80°$, $a = 9$, $b = 2$

SOLUTION:
Use the Law of Cosines to find the missing side length.

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos C \\ c^2 &= 9^2 + 2^2 - 2(9)(2)\cos 80^\circ \\ c^2 &\approx 78.7 \\ c &\approx 8.9 \end{aligned}$$

Use the Law of Sines to find a missing angle measure.

$$\begin{aligned} \frac{\sin B}{2} &\approx \frac{\sin 80^\circ}{8.9} \\ \sin B &\approx \frac{2\sin 80^\circ}{8.9} \\ B &\approx 13^\circ \\ m\angle A &\approx 180^\circ - (80^\circ + 13^\circ) \\ &\approx 87^\circ \end{aligned}$$

Find the measure of $\angle A$.

$$\begin{aligned} m\angle A &\approx 180^\circ - (80^\circ + 13^\circ) \\ &\approx 87^\circ \end{aligned}$$

16. $w = 20, x = 13, y = 12$

SOLUTION:

Use the Law of Cosines to find the measure of the largest angle, $\angle W$.

$$w^2 = x^2 + y^2 - 2xy\cos W$$
$$20^2 = 13^2 + 12^2 - 2(13)(12)\cos W$$
$$\frac{20^2 - 13^2 - 12^2}{-2(13)(12)} = \cos W$$
$$106^\circ \approx W$$

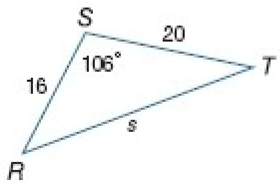
Use the Law of Sines to find the measure of angle, $\angle X$.

$$\frac{\sin 106^\circ}{20} \approx \frac{\sin X}{13}$$
$$\sin X \approx \frac{13\sin 106^\circ}{20}$$
$$X \approx 39^\circ$$

Find the measure of $\angle Y$.

$$m\angle Y \approx 180^\circ - (39^\circ + 106^\circ)$$
$$\approx 35^\circ$$

Determine whether each triangle should be solved by beginning with the Law of *Sines* or the Law of *Cosines*. Then solve the triangle.

18. 

SOLUTION:

Since two sides and their included angle of a triangle are given, the triangle should be solved by beginning with the Law of Cosines.

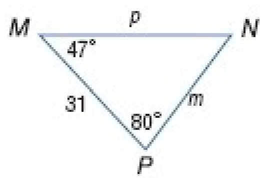
$$s^2 = r^2 + t^2 - 2rt\cos S$$
$$s^2 = 20^2 + 16^2 - 2(20)(16)\cos 106^\circ$$
$$s^2 \approx 832.4$$
$$s \approx 28.9$$

Use the Law of Sines to find a missing angle measure.

$$\frac{\sin R}{20} \approx \frac{\sin 106^\circ}{28.9}$$
$$\sin R \approx \frac{20\sin 106^\circ}{28.9}$$
$$R \approx 42^\circ$$

Find the measure of $\angle T$.

$$m\angle T \approx 180^\circ - (106^\circ + 42^\circ)$$
$$\approx 32^\circ$$

20. 

SOLUTION:

Since two angles and any side of a triangle are given, the triangle should be solved by beginning with the Law of Sines.

Find the measure of $\angle N$.

$$m\angle N = 180^\circ - (47^\circ + 80^\circ)$$
$$= 53^\circ$$

Find the length of m and p .

$$\frac{\sin 53^\circ}{31} = \frac{\sin 47^\circ}{m}$$
$$m = \frac{31 \sin 47^\circ}{\sin 53^\circ}$$
$$m \approx 28.4$$
$$\frac{\sin 53^\circ}{31} = \frac{\sin 80^\circ}{p}$$
$$p = \frac{31 \sin 80^\circ}{\sin 53^\circ}$$
$$p \approx 38.2$$

22. In ΔHJK, h = 18, j = 10, and k = 23.

SOLUTION:

Since three sides of a triangle are given, the triangle should be solved by beginning with the Law of Cosines.

Find the measure of the largest angle, $\angle K$.

$$k^2 = h^2 + j^2 - 2hj\cos K$$
$$23^2 = 18^2 + 10^2 - 2(18)(10)\cos K$$
$$\frac{23^2 - 18^2 - 10^2}{-2(18)(10)} = \cos K$$
$$107^\circ \approx K$$

Use the Law of Sines to find the measure of angle $\angle H$.

$$\frac{\sin 107^\circ}{23} \approx \frac{\sin H}{18}$$
$$\sin H \approx \frac{18 \sin 107^\circ}{23}$$
$$H \approx 48^\circ$$

Find the measure of $\angle J$.

$$m\angle J \approx 180^\circ - (107^\circ + 48^\circ)$$
$$\approx 25^\circ$$

24. **GEOMETRY** A parallelogram has side lengths 8 centimeters and 12 centimeters. One angle between them measures 42°. To the nearest tenth, what is the length of the shorter diagonal?

SOLUTION:

Use the Law of Cosines to find the shorter diagonal.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 8^2 + 12^2 - 2(8)(12)\cos 42^\circ$$
$$a^2 \approx 65.3162$$
$$a \approx 8.1$$

The length of the shorter diagonal is 8.1 cm.

26. **CCSS MODELING** A triangular plot of farm land measures 0.9 by 0.5 by 1.25 miles.

a. If the plot of land is fenced on the border, what will be the angles at which the fences of the three sides meet? Round to the nearest degree.

b. What is the area of the plot of land?

SOLUTION:

a. Let a = 1.25, b = 0.9 and c = 0.5.

Use the Law of Cosines to find the measure of the largest angle.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ 1.25^2 &= 0.9^2 + 0.5^2 - 2(0.9)(0.5)\cos A \\ \frac{1.25^2 - 0.9^2 - 0.5^2}{-2(0.9)(0.5)} &= \cos A \\ 124^\circ &\approx A \end{aligned}$$

Use the Law of Sines to find the measure of $\angle B$.

$$\begin{aligned} \frac{\sin 124^\circ}{1.25} &\approx \frac{\sin B}{0.9} \\ \sin B &\approx \frac{0.9 \sin 124^\circ}{1.25} \\ B &\approx 37^\circ \end{aligned}$$

Find the measure of $\angle C$.

$$\begin{aligned} m\angle C &\approx 180^\circ - (124^\circ + 37^\circ) \\ &\approx 19^\circ \end{aligned}$$

b. Substitute b = 0.9, c = 0.5 and A = 124º in the area formula.

$$\begin{aligned} \text{Area} &= \frac{1}{2}bc\sin A \\ &= \frac{1}{2}(0.9)(0.5)\sin 124^\circ \\ &\approx 0.19 \text{ mi}^2 \end{aligned}$$

28. **RIDES** Two bumper cars at an amusement park ride collide as shown

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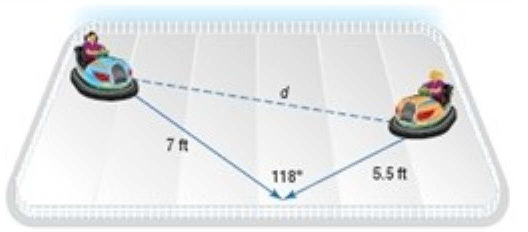


a. How far apart *d* were the two cars before they collided?

b. Before the collision, a third car was 10 feet from car 1 and 13 feet from car 2. Describe the angles formed by cars 1, 2, and 3 before the collision.

SOLUTION:

a. Let *a* = 5.5, *b* = 7, *c* = *d* and *C* = 118

Use the Law of Cosines to find the missing side length.

$$d^2 = 5.5^2 + 7^2 - 2(5.5)(7)\cos 118^\circ$$
$$d^2 \approx 115.399$$
$$d \approx 10.7$$

The distance between the two cars is about 10.7 ft.

b. Let *a* = 13, *b* = 10 and *c* = 10.7

Use the Law of Cosines to find the measure of the largest angle.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$13^2 \approx 10^2 + 10.7^2 - 2(10)(10.7)\cos A$$
$$\frac{13^2 - 10^2 - 10.7^2}{-2(10)(10.7)} = \cos A$$
$$78^\circ \approx A$$

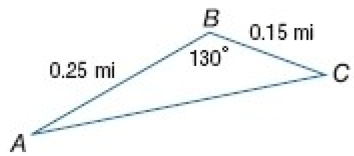
Use the Law of Sines to find the measure of $\angle B$.

$$\frac{\sin 78^\circ}{13} \approx \frac{\sin B}{10}$$
$$\sin B \approx \frac{10\sin 78^\circ}{13}$$
$$B \approx 49^\circ$$

Find the measure of $\angle C$.

$$m\angle C \approx 180 - (78 + 49)$$
$$\approx 53$$

30. **WATERSPORTS** A person on a personal watercraft makes a trip from point *A* to point *B* to point *C* traveling 28 miles per hour. She then returns from point *C* back to her starting point traveling 35 miles per hour. How many minutes did the entire trip take? Round to the nearest tenth.



SOLUTION:
Use the Law of Cosines to find the missing side length.

$$b^2 = 0.15^2 + 0.25^2 - 2(0.15)(0.25)\cos 130$$
$$b^2 \approx 0.1332$$
$$b \approx 0.36 \text{ mi}$$

The time taken for the trip from point *A* to point *B* is $\frac{0.25}{28} \approx 0.0089$ per hour.

The time taken for the trip from point *B* to point *C* is $\frac{0.15}{28} \approx 0.0054$ per hour.

The time taken for the trip from point *C* to point *A* is $\frac{0.36}{35} \approx 0.0103$ per hour.

Time taken for the entire trip is 0.0246 hrs or 1.5 minutes.